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Three essays on regulatory, market, and estimation risk

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Three Essays on

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The Chairman of the Doctoral Board: Prof Dr. Steven Ongena

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The life of an external PhD student engaged in scientific research and at the same time working in the financial industry is more demanding than imagined.

Finding a balance between studies, work, and life has been one of the biggest challenges of my life. The writing of this thesis will, therefore, always be counted as an unforgettable achievement.

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Contents

Part I

Introduction and Summary of Results

1 Introduction

The return of an investment is often positively correlated with the amount of risk taken. In simpler words, there is no return without risk. So, a wise investor might consider including risk as a driver to his investment decisions. But which type of risk should an investor consider? This dissertation focuses on risk-based investment decisions by considering three different types of risk: regulatory risk, market risk, and estimation risk. Regulatory risk, hardly estimable from historical data, reflects how changes in the regulatory environment might impact the profitability of a particular asset class, sector, or even a single company. Market risk emerges from the fluctuations of the market-assigned values of stocks, bonds, commodities, etc. Estimation risk materializes due to natural constraints such as lack of data and a large investment universe at the time of an investment decision.

Three research papers constitute this dissertation, and they are briefly described in the following:

(i) Capital Floors, the Revised SA and the Cost of Loans in Switzerland

(with William Perraudin, and Peng Yang),

(ii) Maximum Diversification Strategies Along Commodity Risk Factors

(with Markus Leippold, and Harald Lohre),

(iii) Second Order Risk In Alternative Risk Parity Strategies

(with Markus Leippold, and Harald Lohre),

2 Summary of Results

(i) Capital Floors, the Revised SA and the Cost of Loans in Switzerland

The first research article of this dissertation looks at regulatory risk, an often neglected source of risk within the process of portfolio construction. This article especially focuses on the revision of the standardized approach to credit risk-weighted assets calculation and also on the possible introduction of a system of capital floors within Switzerland. The regulatory reform is found to produce shifts in levels of capital requirements as well as shifts in capital requirements across banks and asset classes. We find that the reform over-penalizes asset classes like income-producing real estates and corporate lending in Switzerland.

(ii) Maximum Diversification Strategies Along Commodity Risk Factors

The second research article focuses on mitigating market risk within the commodity market. This paper analyses commodity portfolio construction strategies on the basis of uncorrelated risk factor decompositions of the underlying investment universe. Leveraging on the current literature, we propose a new portfolio construction strategy with diversification characteristics similar to the diversified risk parity strategy, with additional benefits of a lower portfolio turnover and a stable risk exposure. This is obtained by equally allocating risk to a portfolio which closely tracks an economically feasible commodity factor model.

(iii) Second Order Risk In Alternative Risk Parity Strategies

The third research article investigates the second-order risk bias, a bias related to estimation risk, in portfolio construction. This bias can be traced in the systematic underestimation of portfolio volatility in samples during portfolio construction. It might materialize especially in modern portfolio theory due to the latters reliance on pure statistical portfolio construction strategies. The minimum variance portfolio, as documented in the literature, is not the only strategy to suffer from this bias. In this paper we show how alternative risk parity strategies also suffer from such bias in their volatility estimation and accordingly propose a new portfolio strategy, known as portfolio risk parity, which is immune to this bias.

Part II

Capital Floors, the Revised SA and the Cost of Loans in Switzerland

Simone Bernardi, William Perraudin, and Peng Yang

This paper was presented at

- UBS A.G. Brown Bag Seminar, Zurich, Switzerland (August 2015)
- Brown Bag Lunch Seminar at University of Zurich, Zurich, Switzerland (April 2016)

Abstract
The Basel Committee plans to revise the Standardised Approach (SA) to bank capital for credit risk and to employ the revised SA as a floor for bank capital based on internal models. The changes are likely to have a major impact on the overall level of capital and its distribution across banks and asset classes. This paper examines the effects of the proposed changes in capital rules on the Swiss loan market. Using primarily public information, we estimate the effects on the capital of individual Swiss banks broken down by asset class. We infer what this is likely to imply for lending rates in the Swiss market. We find that the proposed rule changes would substantially boost capital overall, affecting most severely capital for Corporate and Specialised Lending exposures. Under the BCBS 347 proposals, total bank capital would rise 39% while capital for Corporate and Specialised Lending exposures would increase by 142% and 130%, respectively. This allocation of capital across asset classes is inconsistent with the lessons of the recent financial crisis which was triggered by the collapse of the US residential mortgage market and involved relatively little impact on the quality of corporate credit. By our calculations, bank spreads for corporate loans would rise by between 63 and 103 basis points.

Keywords: Credit Risk, Regulation, Banks, Public Institutions

JEL Classification: G21; G28

1 Introduction

The Basel Committee has recently published proposals for major revisions in an important component of regulatory capital rules, the credit risk Standardised Approach (SA). Under the Committees initial proposals (see Basel Committee on Banking Supervision (2014b), also known as BCBS 307), risk weights for Bank, Corporate and Residential Mortgage exposures would depend on the values of risk indicators, specific to the exposure in question.[1] Recently, the Committee has issued a new version of its proposals retreating from the extensive use of risk indicators (see BCBS (2015), known as BCBS 347). The Basel Committee also published in December 2014 a consultation paper on the use of capital floors (see Basel Committee on Banking Supervision (2014a), also known as BCBS 306). In this, the authorities aim to "tidy up" discrepancies across regulatory jurisdictions in the approach taken to capital floors. When the Basel II rules came into force, regulators applied a temporary capital floor equal to a declining fraction of Basel I capital levels. Following the crisis, this was retained in various forms in different jurisdictions. Since it is imposed at a bank level and is worked out excluding Basel III capital categories such as CVA, in practice, it does not bind many large banks and plays a limited role in pricing decisions.

Regulators regard capital floors as a way of enforcing greater uniformity of risk weight calculations across banks. The Basel Committee has, for some time, expressed dissatisfaction with the inconsistency across banks of capital calculated using internal models (including Internal Ratings-Based Approach (IRBA) credit risk capital calculations). Basel Committee on Banking Supervision (2013), for example, documents such inconsistencies, presenting banks' IRBA risk weight calculations for a set of reference exposures.[2] The authorities have engaged in other policy steps to reduce inconsistencies in capital calculations including an extensive set of evaluation exercises referred to as the Regulatory Consistency Assessment Program (RCAP). The effectiveness of this and parallel industry benchmarking activities in improving consistency has yet to be established. But, the au-

[1] For example, for residential mortgages, the risk indicators that the authorities propose to use as the basis for regulatory capital are Loan to Value (LTV) and Debt Service Coverage (DSC) ratios. More information on the risk indicators may be found in Section 2.

[2] Reportedly, some senior regulators from countries in which the recent crisis had little or no impact have worried about the low default probabilities that banks have estimated and, hence, the low IRBA risk weights that are currently being used.

thorities have decided to push ahead by implementing systematically capital floors based on revised SA rules.

While they have attracted little attention outside risk and regulation specialists, the BCBS 306 and 307 proposals may have far-reaching implications for banks and the economies in which they operate.[3] In particular, the new rules will shift capital between SA and IRB banks and across asset classes. Understanding the nature of these shifts and the economic implications is an important topic of study.[4] TThe Basel Committee organised an official Quantitative Impact Study (QIS) for the BCBS 306 and BCBS 307 version of the proposed new rules but many banks found it difficult to obtain the data necessary to calculate capital accurately. So, the reliability of the QIS, the results of which are in any case confidential, is open to doubt. A new QIS is currently taking place following publication of BCBS 347.

In this paper, we examine the implications of both the 2014 and 2015 versions of the proposals for a particular loan market, that of Switzerland. Primarily using public data, we investigate which banks and asset classes would attract higher or lower capital under the proposed changes. We then proceed to analyse how the changes in capital will affect lending rates. We focus on Swiss banks exposures to Banks, Corporates, Commercial and Residential Mortgage and Specialised Lending exposures located in Switzerland. We study the effects of the proposals on the capital and lending rates of 37 group or individual banks. These include the main suppliers of loans in the Swiss market: two large IRBA banks, UBS and Credit Suisse; a large network SA bank, Raiffeisen (which is particularly active in residential mortgage lending); a group of Cantonal banks of varying size (that are all SA with one IRBA exception); and a group of other SA banks.[5] We perform quantitative impact analysis of the proposals using data published by these 37 banks through their Pillar 3 disclosures and financial statements, calculating the implied changes in the capital

[3]It is worth noting that, following the crisis, the Basel Committee adopted major changes to the Basel II (see BCBS (2006)) capital definitions and capital target ratios. But, aside from the area of trading book rules, these changes (see Basel Committee on Banking Supervision (2009a), (2009b) and (2010b)) involved relatively minor changes in the definitions of Risk Weighted Assets (RWAs). The changes proposed in BCBS 307 are the first major post-crisis reform in RWAs.

[4]One may also be concerned that basing regulatory capital on accounting-data-related risk indicators will shift capital between sectors and jurisdictions in ways that depend more on differences in accounting practice than risk. In some countries, difficulties in obtaining the data necessary to calculate the indicators will mean that capital defaults to punitive values.

[5]In our results, we aggregate Raiffeisen with the Other SA Banks.

individual banks apply to different asset classes. The private data we employ consists of estimates, supplied to us by UBS, of the distribution of its lending within Switzerland conditional on credit quality and the revised SA risk drivers. Using the above information, we first perform top-down calculations of how one might expect individual banks risk weights for each of several asset classes to be affected by the introduction of the BCBS 307 and BCBS 347 rules and the BCBS 306 capital floors regime. Second, we analyse the impact of the capital changes on the spreads that banks charge in different sectors of the Swiss loan market. Third, we calculate the immediate, direct monetary cost of the rule changes as the product of spread changes and current volumes. We do this in annual flow terms and also as a discounted sum of future costs.

To infer the impact of increased capital on spreads, we calculate the cost of bank equity employing the Capital Asset Pricing Model (CAPM) suggested by Kashyap, Stein, and Hanson (2010) and subsequently used by Miles, Yang, and Marcheggiano (2012) and Junge and Kugler (2013). This approach yields not just a calculation of the initial cost of equity but also an estimate of how that cost of equity may change as a bank increases its capital. In contrast to these other authors, we examine the impact of capital changes explicitly distinguishing between the costs of equity of individual banks. Our most important finding is that the proposed changes in the capital rules would significantly boost the spreads that banks charge to Corporate and Specialised Lending borrowers. We also conclude that the changes would significantly improve the relatively competitive position of the Cantonal Banks vis-à-vis the two large Swiss banks. The increase in capital and cost of lending to Corporates runs counter to one of the policy lesson of the recent crisis in which corporate loans performed well in many countries while residential mortgages contributed, at least in the US, to major instability. They are also inconsistent with recent concerns voiced by policy-makers in Switzerland about dangers of over-heating in the residential mortgage market.[6]

One may note that the regulatory landscape for Swiss banks is evolving not just because of the rule changes discussed in this paper. Examples of other developments include the phased

[6]For example, OECD (2012) (see page 12) discusses concerns of over-heating in the Swiss housing market. Brown and Guin (2013) examine the sensitivity of Swiss mortgage borrowers to interest rate and house price changes in the light of concerns about the stability of the market expressed by policy-makers. They find that these sensitivities are potentially serious in the long run although less important in the short or medium term. Bourassa, Hoesli, and Scognamiglio (2012) describe features of the Swiss housing market that made it more stable prior to the crisis and hence less subject to price falls afterwards, including the conservative lending practices of Swiss banks.

introduction of Basel leverage ratios, alterations in trading book regulations and the minimum Total Loss Absorbing Capacity (TLAC) rules. Here, we focus on the revised credit risk SA and its interaction with proposed capital floors since these changes have attracted relatively little attention and yet have the potential to alter very substantially the distribution and level of bank capital.

This paper is a contribution to a substantial literature on the impact of alterations in regulatory capital rules on aggregate bank capital and the wider economy. Repullo and Suarez (2004) and Ruthenberg and Landskroner (2008) examine the effects of the introduction of the Basel II rules on lending rates, focusing on how a bank's choices between SA and IRBA approaches would affect outcomes. Recent papers by Elliott (2009), King (2010), Kashyap, Stein, and Hanson (2010), Basel Committee on Banking Supervision (2010a), Macroeconomic Assessment Group (2010), Institute of International Finance (2011), Cosimano and Hakura (2011), Slovik and Cournde (2011), Miles, Yang, and Marcheggiano (2012), Junge and Kugler (2013), Backler and Wurgler (2013) and Basten and Koch (2014) study the economic effects of the increases in capital envisaged in Basel III. Other studies have examined the dynamics of bank lending and capital econometrically. Early studies include Hancock, Laing, and Wilcox (1995), Peek and Rosengren (1995) and Ediz, Michael, and Perraudin (1998). More recent analyzes include Mora and Logan (2010), Francis and Osborne (2012) and Peek, Rosengren, and Tootell (2011). For other relevant studies see for example Bassett, Chosak, Driscoll, and Zakrajŝek (2010), who examine how bank loan supply shocks feed through into real economic activity. This study may also be viewed as a contribution to the literature on the Swiss banking market. This includes among other significant studies Neuberger and Schacht (2005), Dietrich (2009), Dietrich and Wanzenried (2010), Rochet (2014) discusses studies of the economic impact of capital rules in the context of Swiss bank regulation.

The paper is organised as follows. Section 2 describes the proposal changes in capital rules. Section 3 details how we map the Basel BCBS 306, 307 and 347 proposals into estimates of changes in the capital individual banks will hold against exposures in different asset classes. Section 4 explains how we analyse the impact on spreads, again by bank and asset class. Section 5 presents the results of our calculations. Section 6 concludes. The Appendix contains information on how we estimate risk driver distribution for Swiss bank exposures to other Swiss banks in the context of BCBS 307 rules and the distribution of unrated loans that we employ in implementing BCBS 347 rules.

2 The Revised SA and Capital Floors

2.1 Background

This paper examines the impact on the Swiss loan market of the proposed changes in bank capital rules set out in BCBS 306, 307 and 347.[7] This involves calculating the impact on capital for differentbanks and asset classes and then analysing how this will affect the spreads at which banks lend. We begin by providing background to the Basel Committees proposals.

The existing credit risk SA is employed by banks that choose, subject to regulatory approval, to use less sophisticated approaches to calculating regulatory capital. The SA includes a set of asset-class specific risk weights that banks apply to their exposures to calculate their credit-related Risk Weighted Assets (RWAs). A banks required capital is then calculated by multiplying its total RWA by a capital target ratio. Under Basel I and II rules, banks apply target ratios of 4% and 8%, respectively, to their RWAs to derive their required Tier I and Tier II capital. Under Basel III, the system of capital target ratios is more complex and includes elements based on a Capital Conservation Buffer and a Counter-Cyclical Buffer as well as additional percentages for Systemically Important Financial Institutions (SIFIs). Risk weights in the existing credit risk SA are relatively insensitive to risk in that they vary across, but not within, broad asset classes. Exceptions are exposures to rated corporate, bank or sovereign borrowers for which risk weights are determined, based on the exposures credit rating, using look-up tables.

When Basel II was introduced, in order to prevent a possible, sudden reduction in capital levels for some institutions, a Basel I capital floor was included. Under this approach, a bank's required capital equals the maximum of its Basel II level and a percentage of the Basel I level (see BCBS

[7]Basel rule changes like those proposed in BCBS 306 and 307 are rarely subjected to detailed, public analysis. The authorities current approach involves calibration efforts internal to the regulatory community followed by QIS exercises employing data provided by banks. But, the calibration exercises and the results of QIS analysisare rarely disclosed in any detail. Academics have analysed important packages of measures such as Basel III capital changes but their studies are typically performed long after decisions have been made.

(2006) paragraph 45). The Basel Committee intended that the Basel I floor be temporary. It was planned that the percentage used in the floor definition would fall over time from 95% in 2007, to 90% in 2008 and then to 80% in 2009, after which the floor would be dropped. Following the 2007 crisis, however, some jurisdictions decided to maintain the Basel I floor. For example, the European Union determined to retain an 80% Basel I floor, at least until 2017 (see Article 500 of the Credit Risk Regulation (CRR) in European Parliament (2013)).[8] Switzerland also retained the Basel I floor after 2009. The fact that the Basel I floor operates on total bank capital and excludes important new Basel III capital components (such as CVA-related capital) means that for large banks, the Basel I floor does not bind and plays a limited role in banks' loan pricing decisions.

2.2 The BCBS 307 risk weights

Key elements of BCBS 307 that are material to our analysis are the risk weight look-up tables for exposures in individual asset classes. While the existing SA bases risk weights on agency ratings (where available) or employs simple undifferentiated risk weights for wide classes of exposures, under the revised SA, the Basel authorities propose in BCBS 307 to calculate risk weights on the basis of risk indicators consisting of financial ratios. For Bank Exposures, the risk indicators are the Core Equity Tier 1 ratio of the counter-party bank and the ratio of Net Non-Performing Assets to total loans. Table 1 shows the risk weights, proposed in BCBS 307, for exposures that have CET1 and NNPA ratios in particular, specified ranges. One may observe that the risk weights range from 30% to 300%, a substantial "times 10" range from least to most risky banks.

[8] Even when jurisdictions operate a Basel I floor, they may do so in different ways. In the European CRR formulation of the floor (see European Parliament (2013), Basel II capital must exceed a percentage of Basel I capital. In contrast, Basel Committee on Banking Supervision (2006) envisages that Basel II risk weights exceed a percentage of Basel I risk weights. Borchgrevink (2012) shows, through examples, that floors based on capital levels are markedly less conservative than floors based on risk weights.

Table 1
RSA risk weights for bank exposures

The table, reproduced from BCBS 307, shows the risk weights banks must use for exposures to other banks under the revised credit risk SA. The risk weights depend on the Common Equity Tier 1 (CET1) ratio and Net Non-Performing Asset (NNPA) ratio of the bank in question.

			NNPA ratio $(.) \leq \mathbf{1}\%$	**NNPA ratio** $\mathbf{1}\% < (.) \leq \mathbf{3}\%$	**NNPA ratio** $\mathbf{3}\% < (.)$
	CET1 ratio	$\geq 12\%$	30%	45%	60%
$12\% >$	CET1 ratio	$\geq 9.5\%$	40%	60%	80%
$9.5\% >$	CET1 ratio	$\geq 7\%$	60%	80%	100%
$7\% >$	CET1 ratio	$\geq 5.5\%$	80%	100%	120%
$5.5\% >$	CET1 ratio	$\geq 4.5\%$	100%	120%	140%
	CET1 ratio	$< 4.5\%$	300%	300%	300%

If the data required for a bank to calculate capital for an exposure to another bank on this basis is not available (for example, because the obligor bank does not possess Basel III consistent RWA data and, hence, cannot publish a CET1 ratio), the default risk weight value is 300%. This approach contrasts with the current SA in which if a rating is not available, risk weights equal the Basel I level of 100%. For Corporate Loans, the capital indicators proposed in BCBS 307 are Revenue and a Leverage ratio (defined as total assets over common equity). Table 2 shows the risk weights for different risk indicator ranges. In this case, proposed risk weights range from 60% to 300%, i.e., a "5 times" proportional variation. Leverage is a particularly controversial indicator to use since it varies so much across sectors without corresponding observed variation in default rates and loss given default.

Table 2
Risk weights for corporate exposures

The table, reproduced from BCBS 307, shows the risk weights banks must use for exposures to corporates under the revised credit risk SA. The risk weights depend on the obligor's leverage (the total liabilities to equity ratio) and on gross revenue.

	Revenue $(.) \leq$ **5m**	**Revenue** **5m** $<(.) \leq$ **50m**	**Revenue** **50m** $<(.) \leq$ **1bn**	**Revenue** $(.) >$ **1m**
Leverage $1x - 3x$	100%	90%	80%	60%
Leverage $3x - 5x$	110%	100%	90%	70%
Leverage $> 5x$	130%	120%	110%	90%
Negative Equity (*)	300%	300%	300%	300%

Tables 3 and 4 show the risk weights, proposed by the Basel authorities, for exposures to Commercial and Residential Mortgages. The risk weights in both cases depend on Loan to Value (LTV) ratios while Residential Mortgage risk weights also depend on Debt Service Coverage ratios.

Table 3
RSA risk weights for commercial mortgages

The table, reproduced from BCBS 307, shows the risk weights banks must hold, under the revised credit risk SA against exposures to commercial mortgages. Risk weights depend on Loan to Value (LTV) ratios.

LTV $< 60\%$	$60\% \leq$ **LTV** $< 75\%$	$75\% \leq$ **LTV**
75%	100%	120%

The revised SA further defines so called Specialised Lending exposures. These are exposure types deemed to be particularly risky and are subject to a conservative non-risk-differentiated risk weight. Among others, Income Producing Real Estate (IPRE), Commodity Trade Finance (CTF) and Land Acquisition (LA) given certain conditions might qualify as Specialised Lending exposures, receiving 120%, 120% and 150% risk weights, respectively.

Table 4
RSA risk weights for residential mortgages

The table, reproduced from BCBS 307, shows the risk weights banks must hold, under the revised credit risk SA against exposures to residential mortgages. Risk weights depend on Loan to Value (LTV) and Debt Service Coverage (DSC) ratios.

			DSC $\leq$ 35%	**DSC > 35%**
	LTV	< 40%	25%	30%
40% $\leq$	LTV	< 60%	30%	40%
60% $\leq$	LTV	< 80%	40%	50%
80% $\leq$	LTV	< 90%	50%	70%
90% $\leq$	LTV	< 100%	60%	80%
100% $\leq$	LTV	< 4.5%	80%	100%

2.3 The BCBS 347 risk weights

In this section, we describe the changes that the Basel authorities made to their revised credit risk SA proposals in BCBS 347 following a hostile industry response to BCBS 307. We begin with risk weights for exposures to Banks. As explained above, under the BCBS 307 proposals, banks determined risk weights for their exposures to other banks based on the obligor's CET1 ratio and net non-performing assets (NNPA) ratio. Most respondents to the Committees consultation accepted the use of the CET1 ratio but many argued the NNPA ratio was not comparable across different accounting regimes. Some thought that the two-risk driver approach was overly simplistic and would result in a loss of risk information and others pointed out the elimination of dependence on ratings was unnecessary and undesirable. In its BCBS 347 revision, the Committee acknowledged the limitations of BCBS 307 and proposed that bank exposures be risk-weighted based on the following hierarchy.

a. External Credit Risk Assessment Approach (ECRA)

Banks incorporated in jurisdictions that allow the use of external ratings for regulatory purposes would assign to their rated bank exposures the corresponding Base risk weights depending upon the external ratings as shown in Table 1. Bank exposures with maturity of three months or less could be assigned a risk weight based on the second row in Table 5.

Table 5
Risk weights for bank exposure (ECRA)

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to other banks. Risk weights depend on the availability of external ratings and the tenor of the exposure.

External rating	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to B-	Below B-
"Base" risk weight	20%	50%	50%	100%	150%
Rw. short term	20%	20%	20%	50%	150%

b. Standardised Credit Risk Assessment Approach (SCRA)

Banks incorporated in jurisdictions that allow the use of external ratings for regulatory purposes would classify their unrated bank exposures into one of three risk-weight buckets: Grades A, B and C, using an approach termed the Standardised Credit Risk Assessment Approach (SCRA). Banks incorporated in jurisdictions that do not permit use of external ratings for regulatory purposes would apply the SCRA approach to all their bank exposures. Risk weights for SCRA are according to Table 6.

Table 6
Risk weights for bank exposure (SCRA)

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to other banks. Risk weights depend on the credit risk assessment and the tenor of the exposure.

Credit risk assessment	Grade A	Grade B	Grade C
"Base" risk weight	50%	100%	150%
Risk weight for short term exposure	20%	50%	150%

We now turn to risk weights for Corporate exposures. In BCBS 307, as for Bank exposures, Corporate-exposure risk weights are determined by a two-risk-driver approach, specifically revenue and leverage. Respondents to the Committees consultation deemed the use of leverage inappropriate without consideration of a corporates industry sector while the use of revenue was also criticised as it would penalise SMEs. In response, in BCBS 347, the Committee proposes two approaches to apply the risk weights for corporate exposures.

a. For banks incorporated in jurisdictions that allow the use of external ratings for regulatory purposes, the risk weights of corporate exposures will be determined according to Table 7.

Table 7
Risk weights for corporate exposure

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to corporates. Risk weights depend on the availability of external ratings.

External rating of counterparty	**AAA to AA-**	**A+ to A-**	**BBB+ to BBB-**	**BB+ to B-**	**Below B-**	**Unrated**
"Base" risk weight	20%	50%	100%	100%	150%	100%

Unrated corporate SMEs would be assigned an 85% risk weight.

b. For banks incorporated in jurisdictions that don't allow for external ratings for regulatory purposes, banks will apply an 75% risk weight to all "investment grade" corporate exposures and an 100% risk weight to all other corporate exposures.

For residential real estate, BCBS 307 proposed determining risk weights from two risk drivers: the loan-to-value (LTV) ratio and the debt servicing coverage (DSC). The use of LTV ratio was generally supported by respondents to the Committee's consultation but they expressed significant concerns on the use of the DSC ratio due to the challenges of defining and calibrating the DSC ratio. In BCBS 347, the Committee, therefore, decides to retain the LTV ratio as the risk driver but not to use the DSC ratio (see Table 8). The risk weights would vary based on the exposures' LTV ratio and also would depend on whether repayment is materially dependent on cash flows generated by the property (see Table 8 and Table 9).

Table 8
Risk weight for residential real estate exposures (when repayment is not materially dependent on cash flows generated by property)

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to residential mortgages. This table applies when the repayment of the loan is not materially dependent on cash flows generated by property. For residential real estate exposures to individuals with an LTV ratio higher than 100% the risk weight applied will be 75%. For residential real estate exposures to SMEs with an LTV ratio higher than 100% the risk weight applied will be 85%.

	LTV $\leq 40\%$	$40\% <$ **LTV** $\leq 60\%$	$60\% <$ **LTV** $\leq 80\%$	$80\% <$ **LTV** $\leq 90\%$	$90\% <$ **LTV** $\leq 100\%$	**LTV** $> 100\%$
Risk weight	25%	30%	35%	45%	55%	RW counterparty

Table 9
Risk weight for residential real estate exposures (Repayment is materially dependent on cash flows generated by property)

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to residential mortgages. This table applies when the repayment of the loan is materially dependent on cash flows generated by property.

	LTV $\leq 60\%$	$60\% <$ **LTV** $\leq 80\%$	**LTV** $> 80\%$
Risk weight	70%	90%	120%

For commercial real estate exposures, to ensure consistency with residential real estate exposures, the Committee proposes in BCBS 347 to assign risk weights based on the LTV ratio and on whether repayment is materially dependent on cash flows generated by the property (see Table 10 and Table 11).

Table 10
Risk weight for commercial real estate exposures (Repayment is not materially dependent on cash flows generated by property)

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to commercial mortgages. This table applies when the repayment of the loan is not materially dependent on cash flows generated by property. For commercial real estate exposures to individuals with an LTV ratio higher than 60% the risk weight applied will be 75%. For residential real estate exposures to SMEs with an LTV ratio higher than 60% the risk weight applied will be 85%.

	LTV $\leq 60\%$	**LTV** $> 60\%$
Risk weight	min(60%,RW of counterparty)	RW of counterparty

Table 11
Risk weight for commercial real estate exposures (Repayment is materially dependent on cash flows generated by property)

The table, reproduced from BCBS 347, shows the risk weights banks must hold, under the revised credit risk SA against exposures to commercial mortgages. This table applies when the repayment of the loan is materially dependent on cash flows generated by property.

	LTV $\leq 60\%$	**LTV** $\leq 80\%$	**LTV** $> 80\%$
Risk weight	80%	100%	130%

Last, for Specialised Lending exposures, BCBS 307 proposed to employ the following risk weights:

a. 120% to exposures against project finance, object finance, commodities finance and income-producing real estate (IPRE) finance

b. 150% to exposures against land acquisition, development and construction(ADC) finance

In BCBS 347, to be consistent with the reintroduction of external ratings for risk-weighting exposures to banks and corporate, the Committee proposes to permit use of issue-specific external ratings for project finance, object finance and commodities finance. The applicable risk weight would be determined by the same risk-weight look-up table that would apply to general corporate exposures. The Committee also proposes to categorise IPRE exposures ADC exposures as real estate exposures. IPRE will be treated as real estate exposures with repayment materially dependent on cash flows generated by property. IPRE will either use risk weight look-up Table 9 or Table 11 depending on the sub-category to which it belongs. ADC exposures would still be risk-weighted at 150% but now would include loans to companies and individuals that are made to finance the acquisition of finished property, where the repayment of the loan depends on the future uncertain sale of the property. (We do not reflect this definition change in our calculations.)

2.4 Off balance sheet exposures in BSBC 307 and BCBS 347

An important aspect of the rule changes proposed in the revised credit risk SA concerns the Credit Conversion Factors (CCFs) used for undrawn loan facilities. CCFs are used within the Basel system to convert off-balance sheet exposures of various types to exposures at default comparable to those of conventional drawn loans. BCBS 307 proposed to introduce a CCF of 10% for unconditionally cancellable loan commitments. Previously, they had carried as CCF of zero. For the asset classes we study in this paper, banks have generally regarded undrawn loan commitments as unconditionally cancellable (UCC). The BCBS 307 proposed change therefore represented a small but possibly significant increase in conservatism affecting SA banks directly and IRB banks because of the proposed regime of SA-capital-based floors. BCBS 347, however, proposes a much more important increase in conservatism in that the Committee proposes to apply a reduced CCF between 10% and 20% only to retail commitments (e.g., credit cards). All other non-retail commitments that are currently categorised as UCC would be treated as general commitments. These latter, which would include undrawn loan facilities in the asset classes we study in this paper, would be subject to a CCF of 50% to 75%, the precise calibration to be established in the future. Clearly, the impact of this change both directly for SA banks and, indirectly for IRB banks via the proposed SA-capital-floors regime, would be substantial.

3 Capital Impact Analysis

3.1 Breakdown of the Swiss loan market

This section describes how we infer the impact of the proposed capital rule changes in BCBS 306, 307 and 347 for different banks and asset classes. Table 12 shows the market shares that different categories of bank contribute to the main segments of the Swiss loan market. The pie charts that appear in Figure 1 exhibit the same data as Table 12. One may observe that 70% of Corporate Financing is supplied by the two Large Banks and the Cantonal Banks, the two categories of bank providing roughly equal market shares. The Cantonal Banks have the largest share of the market in Mortgages to Corporates followed by the Other Banks. The largest share of Mortgages to Households is supplied by the Other Banks (which include Raiffeisen), followed by the Cantonal Banks.

Table 12
Swiss credit market volume shares by bank category

Figures displayed are in CHF Million and pertain to the end of 2014. The data source is Swiss National Bank (SNB) reports.[9]

	Banks	Corporate Financing	Motrgage to Corporate	Motrgage to Households	Total Motrgage
Large Banks	9,167	48,112	59,211	197,369	256,580
Cantonal Banks	10,360	45,274	95,645	220,358	316,003
Other	68,535	40,965	66,512	257,584	324,096
All banks	88,062	134,351	221,368	675,311	896,679

The pie charts that appear in Figure 1 exhibit the same data as Table 12.[10]

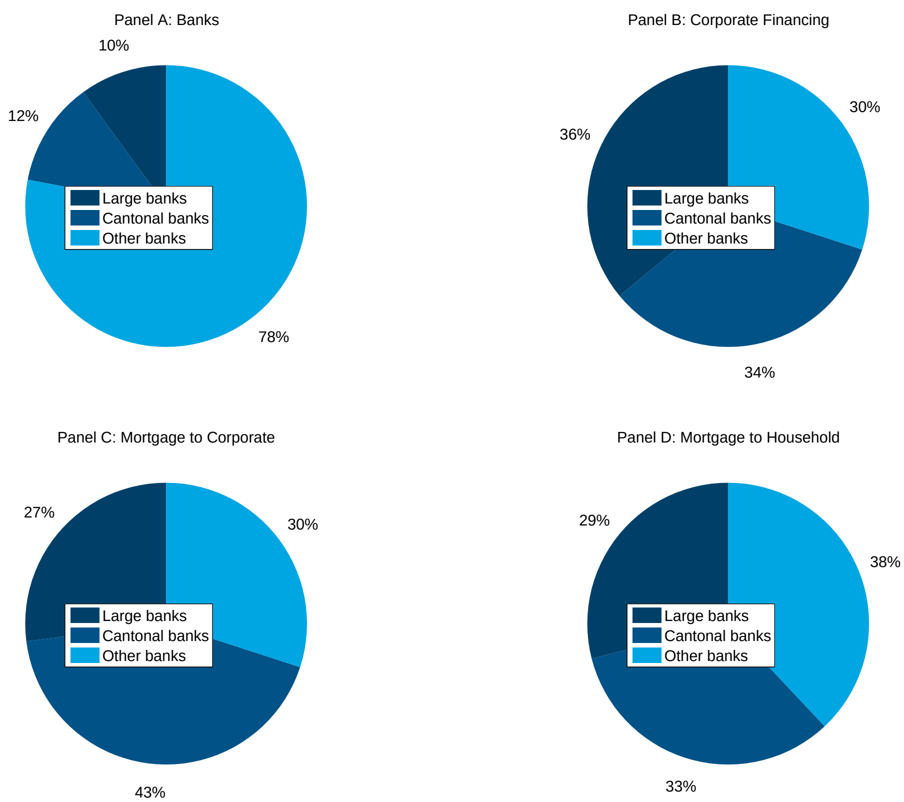
We wish to analyze bank loan exposure data in a disaggregated way.[11] It is natural to work with the standard regulatory categories such as: Sovereign, Bank, Corporate, Other Wholesale, Retail Mortgage, Revolving Facilities and Other Retail. It is not practical, however, to examine all of these categories because of data availability. We, therefore, focus our investigation on capital

[10]The data sources for Table 12 are as follows: The total domestic credit volume in Switzerland, as of December 2014, is CHF 1,066,136 million. The data source is the SNB report: Credit volume statistics domestic and foreign available at http://www.snb.ch/ext/stats/bstamon/pdf/deen/Kreditstatistik_IABG.pdf. Figures on Exposure to Banks come from the SNB report: http://www.snb.ch/ext/stats/bstamon/pdf/deen/Aktiven_I.pdf. Figures on Total Mortgages also come from this report. As this report presents statistics for the total domestic and foreign credit volumes, there is no breakdown by bank groups for domestic credit volume. We assume all foreign lending is performed by Large Banks. Figures on Corporate Financing and Mortgages to Corporate come from SNB report: Credit volume statistics domestic, to companies, by company size and type of loan. http://www.snb.ch/ext/stats/bstamon/pdf/deen/Kreditstatistik_Betriebsgroessen.pdf. Figures on Mortgages to Households are calculated as the difference between Total Mortgages and Mortgages to Corporates.

[11]It is particularly interesting to look at the effects of capital requirements broken down by loan type. Brun, Fraisse, and Thesmar (2013) go even further by using loan level data to examine the effects of capital regulations on lending. They find strong results of capital changes on lending.

and spread impacts for the four key regulatory asset classes: Bank Exposures, Corporate Loans, Commercial Mortgages and Residential Mortgages. In the case of IRB banks, we will also provide some results on the impacts on several categories of Specialized Lending.[12]

Figure 1. Market shares of credit volume of banks in Swiss
Figures displayed are in CHF Million and pertain to the end of 2014. The data source is Swiss National Bank (SNB) reports.



To obtain accurate estimates of impacts on capital, it is necessary to break the loan volumes down further, distinguishing loan exposure data based on (i) the approach the bank uses in calculating regulatory capital (IRB, SA, SRW and Other) and (ii) default probabilities (in the case of IRBA loans) or risk weight bands (in the case of SA loans). We concentrate our analysis only on IRBA and SA loans.

[12]Lack of detailed data on Specialized Lending for Credit Suisse obliges us to make the simplifying assumptions that the bank's exposure to the Specialized Lending category Income Producing Real Estate is the same as UBS, i.e., CHF 20billion.

Table 13
List of banks covered in our study

The table displays the list of banks for which we analyse credit risk exposures to Bank, Corporate and Mortgage Borrowers. The banks are categorised as Large banks, Cantonal banks and Other. The right hand column provides information on whether the Pillar 3 Disclosures or Annual Report of the bank in question contains break downs of credit exposures by PDs or risk weights.

Bank names	**Bank groups**	**RW available**
Credit Suisse	Large banks	YES
UBS	Large banks	YES
Reiffeisen	Other	YES
Aargauische Kantonalbank	Cantonal banks	YES
Appenzeller Kantonalbank	Cantonal banks	NO
Banca dello Stato del Cantone Ticino	Cantonal banks	YES
Banque Cantonale de Genève	Cantonal banks	YES
Banque Cantonale du Jura	Cantonal banks	NO
Banque Cantonale Neuchâteloise	Cantonal banks	YES
Banque Cantonale Vaudoise	Cantonal banks	YES
Basellandschaftliche Kantonalbank	Cantonal banks	YES
Basler Kantonalbank	Cantonal banks	YES
Berner Kantonalbank	Cantonal banks	YES
Freiburger Kantonalbank	Cantonal banks	YES
Glarner Kantonalbank	Cantonal banks	NO
Graubundner Kantonalbank	Cantonal banks	YES
Luzerner Kantonalbank	Cantonal banks	YES
Nidwaldner Kantonalbank	Cantonal banks	NO
Obwaldner Kantonalbank	Cantonal banks	NO
Schaffauser Kantonalbank	Cantonal banks	YES
Schwyzer Kantonalbank	Cantonal banks	YES
St. Galler Kantonalbank	Cantonal banks	YES
Thurgauer Kantonalbank	Cantonal banks	YES
Urner Kantonalbank	Cantonal banks	NO
Walliser Kantonalbank	Cantonal banks	YES
Zuger Kantonalbank	Cantonal banks	YES
Zurcher Kantonalbank	Cantonal banks	YES
Bank J. Safra Sarasin	Other	NO
Bank Linth	Other	YES
Cembra Money Bank	Other	NO
Clientis	Other	YES
Coop Bank	Other	YES
Julius Baer	Other	YES
Migros Bank	Other	YES
Neue Aargauer Bank	Other	NO
Valiant Holding	Other	YES
WIR	Other	NO

While helpful in showing the overall breakdown of the Swiss loan market in a timely fashion (the data we exhibit is for end 2014), SNB data are not sufficiently disaggregated for us to employ directly in our analysis.[13] We, therefore, use individual bank data taken from the annual reports and Pillar 3 disclosures of individual banks. The banks that we study (37 in number) are listed in Table 6. Of these, three are IRB banks, namely Credit Suisse, Banque Cantonale Vaudoise and UBS. Based on statements contained in either the bank's annual report or Pillar 3 disclosures, we consider all other banks to be following the SA in calculating credit risk capital.[14]

3.2 Calculating BCBS 307 revised credit risk SA risk weights

The data we obtain from annual reports and Pillar 3 disclosures pertains to end 2013. To bring the data up to date, we rescale[15] the exposure data so it is consistent with the more timely, end-2014 information in the SNB statistics displayed in Table 12. The rescaled individual bank level loan volume data are displayed (in aggregated form) in Table 14. Because of the re-scaling, they, of course, differ from those published in the banks' 2013 annual reports and Pillar 3 disclosures.

[13] Aggregate statistics on the Swiss banking sector and loan markets may be found in Swiss National Bank (2012) and (2013).

[14] The approach used by Basler Kantonalbank is unclear but we assume it primarily uses the SA. We are aware that some other banks in Switzerland have IRB status at least for some aspects of their capital calculations. There do not appear to be public disclosures that would permit us to allow for this in our study and it may be that the banks in question do not use IRB approaches for the asset classes we consider here.

[15] We rescale the exposure amounts for banks other than the two largest banks (for which we have timely data) to yield totals for the Raiffeisen and Cantonal Banks that equal those reported for end 2014 by the SNB. For Raiffeisen, we aggregate the exposure amount for each asset class; we rescale the total exposure amount for each asset class to match the SNB figures in Table 12. We are only able to rescale mortgages at the level of total mortgages. Since our data on 37 banks does not cover all banks, we create two additional bank groups to represent cantonal banks and other banks which are not covered in the 37 banks. The exposure amounts for the additional cantonal banks group is calculated as the difference between the figures in Table 12 and the aggregated figures for each asset class for the cantonal banks among our 37 banks. We suppose that their risk weights equal the weighted average of those we derive for the cantonal banks among our 37 banks. For the Other Banks group, we create a group called additional other banks and follow the same logic as for cantonal banks so as to cover all remaining banks. Raiffeisen is grouped together with the Other Banks for the purpose of reporting results after all rescaling is complete.

Table 14
Volume shares based on bank level data after re-scaling

Bank level data is only available for end 2013 except for the two large banks (for which end 2014 is available). We rescale data for all except the two large banks so that the aggregates are consistent with end 2014 aggregate data published by the SNB. The resulting rescaled, bank level data is what we employ in our analysis of the capital impact of the revised credit risk SA. Figures are in CHF millions.

Bank groups	Banks	Corporate Financing	Commercial Mortgage	Residential Mortgage	Total Motrgage
Large Banks	9,167	48,112	72,837	183,743	256,580
Cantonal Banks	10,360	45,274	59,575	256,428	316,003
Other	68,535	40,965	29,525	294,571	324,096
All banks	88,062	134,351	161,938	734,741	896,679

After rescaling, we decompose each banks asset-class-specific exposure data according to the PD or risk weight (if this is available) information contained in the banks Annual Report or Pillar 3 Disclosures. For banks that do not publish default probability or risk weight breakdowns, we assume that the breakdown by risk weights equals the weighted average risk weight breakdown of banks for which the information is available.[16] The right hand column of Table 13 shows whether or not we were obliged to make such risk weight assumptions about a given bank. Using the decomposed data for each bank, we proceed to calculate capital requirements using the revised SA approach. The process involves the following steps.

1. For IRB banks, we infer default probabilities (PDs) from risk weights using the standard Basel formula assuming values of loss given default (LGD) and maturity (MT).
2. For SA banks, we infer ratings from RW according to the look-up tables in the current SA approach.
3. From the inferred ratings, we map the corresponding PD based on a default probability master scale table provided by UBS (see Table 15).
4. We devise two rating buckets: AAA to A- and BBB+ to default.

[16] In the case of Banque Cantonale Vaudoise, a breakdown is provided only for the bank's aggregated category: "banks, corporates and other institutions". We, therefore, assume that the bank and corporate exposures of this bank have the same risk weight breakdown as the aggregate category.

5. For each asset class that depends on two capital indicators, we estimate three joint distributions: one unconditional distribution and two conditional distributions for the above two rating buckets.

6. For each asset class, we associate to each of the exposure categories (broken down by credit quality) a distribution of the two capital indicators conditional on their credit quality.

7. Given the look-up table in the revised SA paper, the indicator distribution and the loan exposure at default, we calculate the risk weighted assets and capital requirement for the loan book.

Table 15
Default probabilities

When a bank reports risk weights for a particular loan book, we infer the implied rating category using the existing SA rules and then deduce a corresponding default probability (PD) using the master scale shown in this table. The master scale was provided by UBS.

Rating	PD	Rating	PD	Rating	PD
AAA	0.02%	$A-$	0.08%	$BB-$	2.70%
$AA+$	0.04%	$BBB+$	0.17%	$B+$	4.60%
AA	0.04%	BBB	0.17%	B	7.75%
$AA-$	0.04%	$BBB-$	0.35%	$B-$	13.00%
$A+$	0.08%	$BB+$	0.63%	Cs	22.00%
A	0.08%	BB	1.00%	$Default$	100.00%

In this process, the distribution of exposures by risk indicator plays a crucial role. One may reflect that a bank can calculate its revised SA capital without loan level information if it knows its total exposure in each regulatory loan class and the fractions of those total exposures that fall into each bucket defined by the risk indicator ranges specified in Tables 1, 2, 3 and 4.

Table 16
Generated joint distribution of CET1 and NNPA

To calculate capital under the revised credit risk SA, for a given bank, we need the breakdown of its exposures according to the risk indicators specified in BCBS 307. For bank exposures, the relevant indicators are Common Equity Tier 1 and Net Non-Performing Asset ratios. This table displays the distributions we employed for estimating capital for bank exposures. The distributions differ for Large, Cantonal and Other banks. The methodology employed in estimating these distributions is described in the Appendix.

	CET1 ratio $\geq$ 12%	**CET1 ratio $\geq$ 9.5%**	**CET1 ratio $\geq$ 7%**	**CET1 ratio $\geq$ 5.5%**	**CET1 ratio $\geq$ 4.5%**	**CET1 ratio $\geq$ 4.5%**
			Large banks			
NNPA ratio $\leq$ 1%	79.64%	1.25%	1.25%	0.00%	0.00%	0.00%
1% < NNPA ratio $\leq$ 3%	13.21%	1.25%	0.00%	0.00%	0.00%	0.00%
NNPA ratio > 3%	2.14%	1.25%	0.00%	0.00%	0.00%	0.00%
			Cantonal banks			
NNPA ratio $\leq$ 1%	81.52%	0.94%	0.94%	0.00%	0.00%	0.00%
1% < NNPA ratio $\leq$ 3%	12.59%	0.94%	0.00%	0.00%	0.00%	0.00%
NNPA ratio > 3%	2.14%	0.94%	0.00%	0.00%	0.00%	0.00%
			Other banks			
NNPA ratio $\leq$ 1%	83.39%	0.63%	0.63%	0.00%	0.00%	0.00%
1% < NNPA ratio $\leq$ 3%	11.96%	0.63%	0.00%	0.00%	0.00%	0.00%
NNPA ratio > 3%	2.14%	0.63%	0.00%	0.00%	0.00%	0.00%

To calculate the revised SA capital for each bank in each regulatory asset class, we, therefore, focus on estimating the distribution of loans in the Swiss market across the buckets defined in the BCBS 307 and BCBS 347 tables. In the case of Swiss bank exposures to other Swiss banks, we estimate this distribution based on a combination of public data and informed by guidance to us from an expert with experience of Swiss interbank exposures. This estimation is described in the Appendix. It leads to the distributions shown in Table 16. Almost all Swiss banks for which we have data fall into the highest CET1 bucket given in Table 1 and one may presume that NNPA ratios are very high. Given a judgment-based assumption of the distribution of Swiss bank lending to other Swiss banks, we infer the fractions that Swiss banks in the categories: Large Banks, Cantonal Banks and Other Banks, will have in each of the risk indicator buckets. These are displayed in Table 16. To calculate the risk weights for individual banks implied by the revised SA, one must take the sum of the products of elements in the relevant block of Table 9 with the risk weights specified in Table 1. Let Nr and Nc be the number of categories corresponding to the row and column risk indicators in the table, then the risk weights for the i^{th} bank are:

$$RW_i = \sum_{r=1}^{Nr} \sum_{c=1}^{Nc} p_{r,c,i} \times rw_{r,c}. \tag{1}$$

3.3 Calculating BCBS 347 revised credit risk SA risk weights

BCBS 347 makes some use of risk indicators (in the case of mortgages) but much less so than BCBS 307. Here, we set out the assumptions we make in inferring bank risk weights and capital for Swiss banks under the BCBS 347 rules. We made the following assumptions for unrated bank exposures:

- All large banks are rated
- 50% Cantonal banks and other banks are unrated
- All unrated Cantonal banks are in Grade A
- 70% unrated other banks are in Grade A and 30% unrated other banks are in Grade B

We also made assumptions on interbank credit risk exposure distribution for different bank groups and derived the distribution of unrated bank exposures on bank group level as shown in Appendix 2. We classify corporate exposures with employee size less than 50 as SMEs. We derive the percentage from official SNB statistics report 3Ca.[17]

Table 17
Percentage of SMEs

This table shows the percentage of SMEs consisted in corporate exposures for each bank group. Companies with size less than 50 are classified as SMEs.

Bank group	SME %
Large banks	76%
Cantonal banks	50%
Other banks	71%

In the current SA, the risk weight for unrated exposures is 100%. But in the BCBS 347, the unrated exposures will be risk weighted as either 100% or 85% if they are SMEs. All other risk weighting categories remain the same. We derive the residential mortgage portfolio distribution and commercial mortgage portfolio mortgage distribution from UBS internal portfolio data and

[17]This is available at $http://www.snb.ch/en/iabout/stat/statpub/bstamon/stats/bstamon$.

assume all banks follow the same distributions as UBS. The results of this calculation are shown in Table 18. According to our calculation, the risk weights implied by the BCBS 307 revised SA for the different banks in Switzerland are quite similar, being clustered around 33%. The risk weights implied by the BCBS 347 revised SA for different bank groups vary between 45% and 50%.

Table 18
Revised SA weighted average risk weights

This table shows estimates of Swiss banks risk weights for exposures to other Swiss banks.

	Large banks	**Cantonal banks**	**Other banks**
BCBS 307	53%	55%	49%
BCBS 347	48%	51%	46%

3.4 Off-balance sheet exposure rules

Inspection of SNB statistics indicates that all categories of Swiss banks have extended significant volumes of undrawn loan facilities to Corporate borrowers. Large banks have also provided significant undrawn facilities to Bank counter-parties. On the basis of internal UBS data, we calculate the impacts on Corporate and Bank exposure amounts implied by the BCBS 307 and BCBS 347 proposals for CCFs. In the latter case, the effects lead to an approximate doubling of exposures to both Banks and Corporates. We, therefore, multiple the Corporate exposures of all banks by two and multiple the Bank exposures of the other large bank by two. The Commodity Trading Finance category of Specialised Lending is treated as Corporate category and is boosted by a factor of 2.6 which is derived from UBS actual portfolio data. For UBS, we use our actual estimate of the UBS exposure inclusive of the adjustment for the new CCF rates. For the CCF adjustment under BCBS 307, we follow a similar approach. The scaling factors to boost SA exposure amount are 1.11, 1.16 and 1.2 for Corporates, Banks and CTF respectively.

3.5 Implementing Swiss rules

Swiss banks are required to calculate minimum capital requirements based on using capital target Financial Market Supervisory Authority (FINMA) minimum capital ratios. These are equal to those specified in the Basel III framework plus additional percentages introduced as a so-called Swiss Finish[18]. FINMA minimum capital requirements depend on the size and complexity of banks, divided into 5 categories. Table 19 lists the criteria that determine into which category an institution falls. The institution must meet at least three of the criteria listed to qualify for a given category. Table 12 shows the FINMA minimum capital ratio that banks in each category are required to employ.[19]

In Switzerland, as of end 2014, only four banks have been classified by FINMA as systemically important financial institutions (SIFI) banks and allocated to category 1, and subject to higher minimum capital requirements:[20] UBS, Credit Suisse, Zurcher Kantonalbank (ZKB) and Raiffeisen. SIFIs banks have to hold 10% of total risk weighted assets in CET1 capital (constituted by common shares, retained earnings and other comprehensive income net of regulatory filters and deductions). In addition to CET1 minimum capital requirements, SIFIs have to hold contingent convertible bonds (CoCos), that convert into common equity contingent on the breach of a predetermined ratio of CET1 over total RWA, SIFIs are required to hold a conservation buffer of 3% in form of high trigger CoCos[21], and a progressive component from 1% to 6% of low trigger CoCos.[22]

[18]See FINMA Circular 2011/2.

[19]These measures are expressed as ratios of minimum required capital to total risk weighted assets.

[20]See the Swiss TBTF bank capital regulations.

[21]High trigger CoCos convert to common equity when a 7% ratio of CET1 to total RWA is breached.

[22]Low trigger CoCos convert to common equity when a 5% ratio of CET1 to total RWA is breached.

Table 19
Categorisation of institutions

Swiss rules on capital target ratios differentiate banks based on 5 categories. To qualify for a particular category the scale of a banks activities as measured by at least three of four quantitative indicators must exceed specified thresholds. This table displays the thresholds expressed in CHF millions.

	Total asets	Assets under management	Privileged deposits	Required Equity
Category 1	≥ 250	$\geq 1,000$	≥ 30	≥ 20
Category 2	≥ 100	≥ 500	≥ 20	≥ 2
Category 3	≥ 15	≥ 20	≥ 0.5	≥ 0.25
Category 4	≥ 1	≥ 2	≥ 0.1	≥ 0.05
Category 5	< 1	< 2	< 0.1	< 0.05

Table 20
CET 1 and total capital target

Swiss banks that fall into the categories listed in Table 11 are required to employ the capital target ratios shown in this table. We employ these ratios in our calculations of the capital impact of the revised credit risk SA for Bank and Corporate. For Commercial mortgage exposures and Residential Mortgages, we add an additional 2% reflecting the countercyclical capital buffer adopted by the Swiss authorities for such exposures.

	CET1 capital ratio	Total capital ratio
Category 1	10.0%	14% − 19%
Category 2	8.7% − 9.2%	13.6% − 14.4%
Category 3	7.8%	12.0%
Category 4	7.4%	11.2%
Category 5	7.0%	10.5%

The amount of resolution CoCos a bank must hold depends on the systematic importance of the banks (including total exposure, market share in Switzerland and resolvability considerations). Because of lack of data, the remaining non-SIFI banks were allocated to the FINMA categories 2 to 5 based exclusively on the total asset criterion. Using the assumptions and data described above, one may deduce capital requirement for the i^{th} bank for a given regulatory asset class using the following equation:

$$K_{i,j} = \begin{cases} max(RWA_{i,j}^{target} \times RW_{i,j} \times EAD_{i,j}, LRD_{i,j}^{target} \times EAD_{i,j}) & \text{, if bank } i \text{ is a SIFI} \\ RWA_{i,j}^{target} \times RW_{i,j} \times EAD_{i,j} & \text{, otherwise} \end{cases}$$

Here, $RWA_{i,j}^{target}$ is the risk weight target for the bank in question and $LRD_{i,j}^{target}$ is the leverage ratio target. Under Swiss regulations, the $LRD_{i,j}^{target}$ is equal to $RWA_{i,j}^{target} \times 24\%$. In what follows, we shall focus mainly on the impact of changes in the rules on CET1 capital although we present results below on total regulatory capital as well.

For each SIFI bank i, we adjust the capital by a convexity adjustment ratio which is calculated as follows:

$$ConvAdj_i = \frac{\max\left(\sum_{j=1}^{N} RWA_j \times RWA^{target}, \sum_{j=1}^{N} EAD_j \times LRD^{target}\right)}{\sum_{j=1}^{N} \max\left(RWA_j \times RWA^{target}, EAD_j \times LRD^{target}\right)}. \tag{2}$$

Here, N is the number of asset classes. Such convexity adjustments are implemented in some banks and serve to ensure that the individual exposure class capital amounts add up to total capital once the effects on the latter of both regulatory capital and leverage ratio rules are allowed for. Formula (3.5) may then be modified as follows

$$K_{i,j} = \begin{cases} max(RWA_{i,j}^{target} \times RW_{i,j}, LRD_{i,j}^{target}) \times EAD_{i,j} \times ConvAdj_i & \text{, if bank } i \text{ is a SIFI} \\ RWA_{i,j}^{target} \times RW_{i,j} \times EAD_{i,j} & \text{, otherwise} \end{cases}$$

Up to now, we have concentrated on capital for the exposures of Swiss banks to other Swiss banks. We employ similar approaches to deduce the effect of the revised SA on capital for other asset classes, notably Corporate Loans, Commercial and Residential Mortgages and Specialised Lending. We deduce the corresponding risk weights using the weights for specific risk driver ranges appropriate to Corporate Loans, Commercial and Residential Mortgages, respectively, in Tables 2, 3 and 4. In so doing, we use risk factor distributions based on internal, confidential data supplied by UBS.[23] These distributions consist of the frequencies of loans for the different regulatory risk

[23]Without access to internal bank data, it would be extremely difficult to assess the impact of the revised SA as we do in this paper. To illustrate, even to estimate the distribution of revenue for Swiss SMEs that borrow from banks is very challenging. Summary survey data is available on the average, range and median revenues of such SMEs (CHF

factor buckets with conditional default probability being in certain specified ranges. It is sensible to condition on credit quality in this way because the distribution of loans across risk factors is likely to be very different for high and low credit quality loans. Since we possess data on the default probabilities of loans culled either directly from IRBA default probabilities or inferred from SA risk weights, by conditioning as just described, we are able to obtain a more accurate estimate of the capital impact.

4 Spread Impact Analysis

In this section, we describe how we investigate the spread impact, at an asset class level, for each bank. We assume that:

$$\Delta\text{spread} = \text{capital}_{new} \times \text{return on equity}_{new} \times \frac{\text{RSA EAD}}{\text{Current SA EAD}} - \text{capital}_{old} \times \text{return on equity}_{old}. \tag{3}$$

Here the "capital level" is measured per Swiss franc of exposure. To estimate the return on equity, we use the Capital Asset Pricing Model (CAPM) used in this context by Kashyap, Stein, and Hanson (2010) and by subsequent studies such as Miles, Yang, and Marcheggiano (2012) and Junge and Kugler (2013). This CAPM methodology allows for the possibility that the required return on equity that a bank faces is reduced if its total capital level increases. The required return on equity according to the CAPM equals the net premium on the equity market multiplied by the coefficient of the banks asset return in a regression on an appropriately selected market index. (This net premium on the equity market equals the expected return on the market index minus the return on a short-dated Treasury security.) Thus, for asset class i belonging to bank j, the spread

million 14, 01,450 and 4, respectively) from Christen, Halter, Kammerlander, Knzi, Merki, and Zellweger (2013). But deducing the joint distribution of revenue, leverage and credit quality without private bank data appears impossible. UBS is active throughout Switzerland and in all of the sectors on which we focus. There may be differences between its loan book distribution and that of other individual Swiss banks; but we would expect use of its data to give reasonably unbiased results when one aggregates across multiple banks as we do in our results sections.

change $\Delta spread_{i,j}$ is calculated as:

$$\Delta spread_{i,j} = \begin{cases} \left(K_{i,j}^{RSA} \times \beta_j^{SA} \times \frac{\text{RSA EAD}}{\text{Current SA EAD}} - K_{i,j}^{SA} \times \beta_j^{SA}\right) \times \gamma & , \quad \text{constant cost of equity} \\ \left(K_{i,j}^{RSA} \times \beta_j^{RSA} \times \frac{\text{RSA EAD}}{\text{Current SA EAD}} - K_{i,j}^{SA} \times \beta_j^{SA}\right) \times \gamma & , \quad \text{varying cost of equity} \end{cases}$$

Here, $K_{i,j}^{(\cdot)}$ is the capital requirement per unit exposure amount expressed as:

$$K_{i,j}^{(\cdot)} = \begin{cases} max(RWA_{i,j}^{target} \times RW_{i,j}^{(\cdot)}, LRD_{i,j}^{target}) & , \quad \text{if exposure i belongs to a SIFI} \\ RWA_{i,j}^{target} \times RW_{i,j}^{(\cdot)} & , \quad \text{otherwise} \end{cases}$$

$RW_{i,j}^{(\cdot)}$ is the average risk weight of asset class i in bank j, γ is the equity market risk premium and is set to be 6% in our calculation.[24] β_j is the bank's equity market beta, the regression coefficient of the bank's equity return (net of the safe rate) on a relevant (net) market index equity return. The capital $K_{i,j}^{(\cdot)}$ is then adjusted in the same manner as described in equations (2) and (3.5). We investigate the spread impact using either the CET1 capital target or the total capital target as RWA^{target} in equation (4).

Several past studies have emphasised the possibility that when a bank increases its capital levels, its beta and hence cost of equity funding will fall. This "Modigliani-Miller effect", while indisputably relevant, may be of greater or lesser magnitude and, hence, should be analysed empirically.[25] According to a strict version of the Modigliani-Miller theory (in which banks are viewed as simple and transparent asset pools financed by debt and equity), the banks equity market beta should equal:

$$\beta_{Asset} = \beta_{Equity} \times \frac{Equity}{Assets} + \beta_{Debt} \times \frac{Debt}{Assets}. \tag{4}$$

For simplicity, we suppose that the bank's debt is close to riskless so that: $\beta_{Debt} = 0$, and subtracts from tax effects. In this case, the bank's equity beta will be proportional to the assets-to-equity or

[24] This is consistent with survey evidence from developed economies; see, for example, Fernandez, Linares, and Acin (2014).

[25] Within frictionless markets, the distribution of financing between debt and equity does not affect the discount rate a firm uses to value cash flows. See Modigliani and Miller (1958). For a bank, this implies that lending spreads will be unaffected by holding more equity. When frictions are present such as agency costs, incomplete information or tax differentials between debt and equity, loan spreads may be affected by changes in capital ratios.

"leverage" ratio.

$$\beta_{Asset} = \beta_{Equity} \times \frac{Equity}{Assets}. \tag{5}$$

The above reasoning depends on the absence of frictions such as (i) asymmetries of information between bank insiders and the market, (ii) agency effects in the running of the bank, (iii) asymmetries in the tax treatment of debt and equity. In this sense, it corresponds to an idealized extreme case. To evaluate the empirical magnitude of Modigliani-Miller effects, we allow for a more general dependence of bank beta on leverage in that we suppose:

$$\beta_{Equity} = \alpha_0 + \alpha_1 \times \frac{Assets}{Equity}. \tag{6}$$

Following other authors, we estimate the parameters α_0 and α_1 by (i) estimating betas for a set of banks in different time periods and then (ii) regressing these estimated betas on the leverage level that the relevant bank had at the start of the period in question. There are several important choices that must be made in formulating such regressions. First, one must select an appropriate sample of banks, data frequency, equity index and window length for the beta estimation. Second, having estimated betas, one may choose whether to estimate the relationship between betas and leverage in a fully pooled way or whether to allow for period-specific or bank-specific parameters. Since the regression of beta on leverage has a panel-data form, this latter choice amounts to deciding whether or not to use fixed effects.

Figure 2 shows the log prices of the Swiss banks we covered in regression while Figure 3 shows the Swiss market index. The share prices of the Large Banks and some of the Other Banks appear reasonably correlated with the Swiss equity market index. The Cantonal Bank equity prices on the other hand show little correlation and, indeed, exhibit relatively little volatility. Tables 21 and 22 present results for a range of different equations. Our sample period stretches from 1999 to 2014. The banks included in the estimations are all from Switzerland, the Eurozone or the UK and are chosen on the basis that their assets exceed 10 billion national currency units at the end of the sample period. In all cases, we employ weekly data to estimate the betas. This partly offsets concerns that the equity securities of some banks in the sample might be illiquid. We repeated the exercises using daily data and did not obtain appreciably different results. We estimate betas using data windows one year in length. Again, we verified that the results are not substantially different if a six month window length is employed. The regressions for which we show results in Table 21 vary according to the group of banks analyzed: we exhibit regressions for (a) Swiss banks alone, (b) UK banks alone, (c) Eurozone banks alone and (d) all banks. In each of these four cases, we show results for regressions with no bank or year dummies, with year dummies alone, with banks dummies alone, and with both year and bank dummies. Table 22 shows the same regressions but employing a single European index while the results shown in Table 21 correspond to regressions in which the betas for Swiss, UK and Eurozone banks are measured with respect to Swiss, UK or Eurozone indices, respectively. In all the regressions, the right hand side variables, including the dummies, are demeaned prior to the performing the regression. Hence, the constant in the regression equals the unconditional mean of the left hand side variable in the regression. We will assume, in what follows, that the premium on the equity index is 6%. Since the return on equity equals the product of beta and the premium, we scale the left hand side variable in the regression by 6 so that the constant may be interpreted as the average return on equity across banks implied by the regression expressed in percent.

Figure 2. Selected Swiss banks share prices (in logs)

The figure shows the log share prices of Swiss banks from 1999 to 2015 taken from Bloomberg. Cantonal bank share price time series (apart from that of Banque Cantonale Vaudoise) trend upwards with little volatility suggesting relatively low liquidity. Share prices for the two large banks appear less correlated with those of other banks.

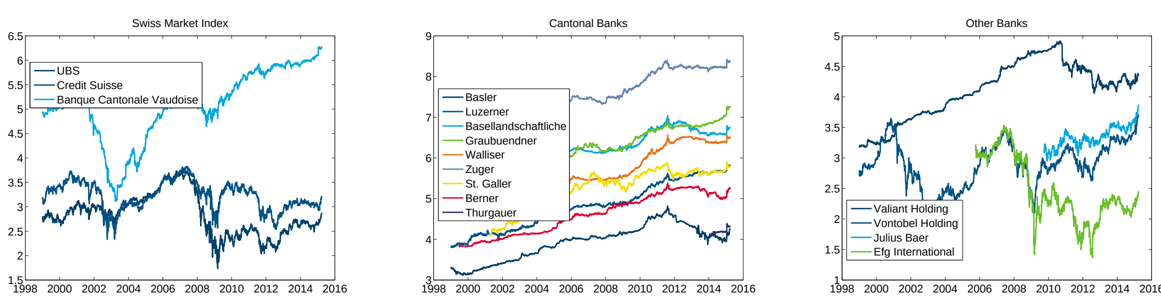
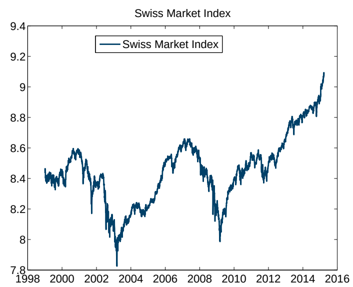


Figure 3. Swiss market index time series

The figure shows the Swiss stock market index from 1999 to 2015 taken from Bloomberg. The index appears correlated with the large bank share prices exhibited in Figure 2 until late in the sample period (post 2011) when the bank share prices under-perform the index.



As one may observe, the average returns on equity implied by the regression constants are low, being 4.3, 7.7, 4.9 and 5.1 percent when national indices are used to estimate betas. Typical returns on equity employed within large European banks are closer to 10%. Inspection of betas for individual banks suggested that there was considerable variation across banks, justifying the use of bank specific dummies in the regression. Examination of the estimates contained in Tables 21 and 22 shows that when bank-specific fixed effects are introduced, the value of the regression coefficient on leverage is significantly reduced. For example, in the case of Swiss banks using betas against a Swiss national index, the leverage coefficient drops from 0.20 to 0.07 when one compares regression 2 (which employs year dummies alone) to regression 4 (which uses both bank and year dummies).[26] It appears likely that the reduction in the size of the leverage effect that occurs when bank dummies are introduced is a reflection of the fact that large banks tend to be more levered and have higher correlation with equity market indices. However, one might reasonably expect that the degree of variation in leverage for individual banks across the sample period should be enough to identify significant leverage effects in required returns on equity if they are present in the data. In the exercises we report below, we will use the estimates corresponding to regression 4 (i.e.,

[26]Backler and Wurgler (2013), like Kashyap, Stein, and Hanson (2010), find strong a relationship between the leverage and equity market betas of US banks. When Baker and Wurgler look only at large institutions involved in investment banking, the results weaken significantly. If returns on investment banks (which tend to be more levered) are more correlated with market indices, then this would exaggerate the apparent relationship between leverage and market betas. Including bank-specific dummies would remove this effect.

including year and bank dummies). This panel data approach seems to us the most defensible given the issues referred to in the last paragraph. The approach is also consistent with that employed in recent studies by Miles, Yang, and Marcheggiano (2012) and Junge and Kugler (2013). We also choose to focus on Swiss banks and to use a Swiss national index. These assumptions appear most sensible given that our study relates to Swiss banks. One might ask why do we find weaker leverage effects than Miles, Yang, and Marcheggiano (2012) and Junge and Kugler (2013)? The latter study employs a log specification of regression. The theory, we would argue is more consistent with the linear specification that we use. In preferring the linear specification, we are consistent with Miles, Yang, and Marcheggiano (2012). When we restrict our data to UK banks and the sample period of Miles, Yang, and Marcheggiano (2012), we obtain results similar to theirs.

Table 21
Beta regression estimates based on weekly return (National index)

The regressions are defined as follows Regression 1: OLS Regression with no bank or year dummies; Regression 2: Fixed effect with year dummy: 2014 dropped; Regression 3: Fixed effect with bank dummy: Walliser Kantonalbank dropped for Swiss banks, Standard Chartered dropped for UK banks, Vseobecna Uverova Banka dropped for Eurozone Banks and European banks. Regression 4: Fixed effect with bank dummy and year dummy: 2014 dropped; Walliser Kantonalbank dropped for Swiss banks, Standard Chartered dropped for UK banks, Vseobecna Uverova Banka dropped for Eurozone Banks and European banks.

	Swiss banks				UK banks			
Regression	1	2	3	4	1	2	3	4
Constant(%)	4.32	4.32	4.32	4.32	7.71	7.71	7.71	7.71
t-statistic	15.34	14.86	24.49	24.55	26.79	36.54	27.69	41.20
Leverage(%)	0.19	0.20	0.08	0.07	0.13	0.08	0.16	0.01
t-statistic	8.80	8.23	2.64	2.28	3.24	2.23	2.86	0.16
R-squared	0.38	0.42	0.79	0.82	0.12	0.62	0.22	0.72
WACC(%)	0.55	0.44	2.84	2.90	5.22	6.22	4.63	7.52
Observations	129	129	129	129	80	80	80	80
	Euro zone banks				European banks			
Regression	1	2	3	4	1	2	3	4
Constant(%)	4.93	4.93	4.93	4.93	5.08	5.08	5.08	5.08
t-statistic	35.85	39.92	44.05	56.63	42.53	45.42	54.70	66.74
Leverage(%)	0.12	0.13	0.04	0.06	0.13	0.14	0.05	0.04
t-statistic	8.34	9.88	1.79	2.82	11.34	12.52	2.78	2.44
R-squared	0.09	0.28	0.45	0.68	0.13	0.25	0.52	0.68
WACC(%)	2.43	2.21	4.01	3.74	2.32	2.15	4.11	4.34
Observations	680	680	680	680	889	889	889	889

Table 22
Beta regression estimates based on weekly return (European index)

The regressions are defined as follows Regression 1: OLS Regression with no bank or year dummies; Regression 2: Fixed effect with year dummy: 2014 dropped; Regression 3: Fixed effect with bank dummy: Walliser Kantonalbank dropped for Swiss banks, Standard Chartered dropped for UK banks, Vseobecna Uverova Banka dropped for Eurozone Banks and European banks. Regression 4: Fixed effect with bank dummy and year dummy: 2014 dropped; Walliser Kantonalbank dropped for Swiss banks, Standard Chartered dropped for UK banks, Vseobecna Uverova Banka dropped for Eurozone Banks and European banks.

	Swiss banks				UK banks			
Regression	1	2	3	4	1	2	3	4
Constant(%)	3.55	3.55	3.55	3.55	7.54	7.54	7.54	7.54
t-statistic	13.39	12.89	23.35	24.35	23.12	33.00	23.94	37.06
Leverage(%)	0.18	0.18	0.08	0.05	0.14	0.07	0.20	0.01
t-statistic	8.81	7.98	3.11	1.86	3.24	1.92	3.22	0.17
R-squared	0.38	0.41	0.82	0.86	0.12	0.65	0.22	0.74
WACC(%)	0.00	-0.01	2.05	2.59	4.73	6.16	3.61	7.33
Observations	129	129	129	129	80	80	80	80
	Euro zone banks				European banks			
Regression	1	2	3	4	1	2	3	4
Constant(%)	5.57	5.57	5.57	5.57	5.44	5.44	5.44	5.44
t-statistic	36.99	40.58	45.86	56.78	42.17	44.45	54.47	64.38
Leverage(%)	0.13	0.15	0.04	0.06	0.14	0.15	0.05	0.04
t-statistic	8.51	9.95	1.41	2.44	11.09	12.08	2.59	2.36
R-squared	0.10	0.27	0.46	0.66	0.12	0.22	0.52	0.66
WACC(%)	2.77	2.53	4.78	4.41	2.52	2.35	4.47	4.65
Observations	680	680	680	680	889	889	889	889

5 Results

In this section, we report the results of our capital and spread impact calculations. We begin by examining the effect of the switch from the current to the revised SA approach for SA banks. Table 23 presents the weighted average risk weights for different asset classes and categories of banks. The weighted averages are worked out using weights based on each individual bank's share of the total exposure of the set of banks being considered. One may observe from Table 23 that the existing weighted average risk weights for all SA banks are 19%, 66%, 92% and 39% for Bank, Corporate, Commercial Mortgage and Residential Mortgage exposures, respectively. There is little variation across the categories of Cantonal and Other banks. Substituting the BCBS 307 revised SA for the existing SA, risk weights change substantially, rising to 120% for Corporate exposures (almost double the existing risk weight level). Bank risk weights are somewhat higher under the revised rather than the existing SA, and, risk weights for Residential Mortgages are actually down from 39% to 37%. Risk weights for Commercial Mortgages drop from 92% to 87%.

Table 23
Current and revised RWs for SA banks

The table shows the risk weights for SA banks under the current SA rules and under the revised credit risk SA rules set out in BCBS 307 and BCBS 347. The aggregated risk weights for each bank category are the weighted average risk weights of individual banks within the category. Results are shown for exposures to counter-parties in Switzerland categorised by Bank exposures, Corporate exposures, Commercial Mortgages and Residential Mortgages.

Bank groups	**Banks**	**Corp. Financing**	**Comm. Mortgage**	**Res. Mortgage**	**Wtd. Avg.**
		Current risk weights			
Cantonal Banks	23%	66%	92%	38%	49%
Other	19%	65%	94%	39%	42%
All SA banks	19%	66%	92%	39%	45%
		BCBS 307 revised SA risk weights			
Cantonal Banks	33%	118%	87%	37%	54%
Other	33%	121%	87%	37%	48%
All SA banks	33%	120%	87%	37%	51%
% change between RSA and SA	73%	82%	−6%	−4%	12%
		BCBS 347 revised SA risk weights			
Cantonal Banks	24%	60%	83%	43%	52%
Other	19%	164%	83%	43%	46%
All SA banks	20%	162%	83%	43%	48%
% change between RSA and SA	3%	−6%	−10%	11%	7%

Substituting the BCBS 347 revised SA for the existing SA, risk weights change much less than they do under the BCBS 307 proposals. As we mentioned in Section 2, for bank exposures and corporate xposures, in BCBS 347, only unrated exposures are treated differently compared to the current SA. Risk weights for Commercial Mortgages drop to 83% while Residential Mortgages increase to 43%.

Figure 4. Weighted average SA bank RW changes

The figure shows percentage changes in risk weights of Swiss SA banks (for selected exposure categories) implied by a switch from the current SA to the revised credit risk SA rules in BCBS 307 and BCBS 347. The exposure categories shown are bank exposures, corporate loans, commercial mortgages and residential mortgages. The figure shows substantial increases in bank exposure and corporate loan risk weights and small declines in mortgage related risk weights.

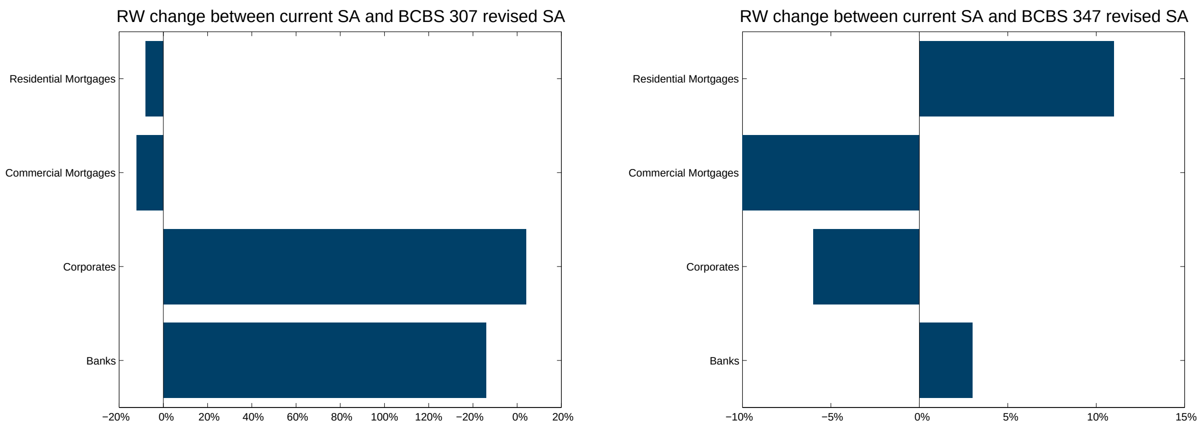


Figure 4 shows the key results from Table 23 in graphical form. Under BCBS 307, Corporate and Bank revised SA risk weights are respectively 82% and 73% higher than the existing SA risk weights, while Commercial Mortgage and Residential Mortgage risk weights are 6% and 4% lower. Under BCBS 347, Bank and Residential Mortgage risk weights are 3% and 11% higher, while Corporate and Commercial Mortgage risk weights are 6% and 10% lower.

Table 24 shows the implied increase in capital that SA banks devote to different segments of the Swiss loan market. Under BCBS 307, the existing CHF 4.3 billion and CHF 1.3 billion capital that SA banks assign to Corporate and Bank lending rises to CHF 8.8 billion and CHF 2.1 billion after the introduction of the revised SA. This is offset by a fall of around CHF 0.22 billion in the capital that Swiss SA banks hold against Commercial and Residential Mortgage lending. Under BCBS 347, there is almost no change in Bank capital. The capital that SA banks hold against Corporate rises to CHF 8.2 billion, however, due to the significant increase in SA exposure amount. The BCBS 347 effects are largely the consequence of the proposal changes in CCF rules. Table 25 shows risk weight calculations results for IRB banks under different scenarios. We present risk weights for the different asset classes and aggregated using exposure-weighted averages (i) under the existing rules, (ii) assuming the revised SA is introduced, (iii) with the revised SA and with 60% exposure-level capital floors, (iv) as in (iii) but with asset-class level floors, and (v) as in (iii) but with a bank level floor. We then repeat scenarios (iii), (iv) and (v) assuming capital floors are imposed equal to 70% and 80% of the revised SA capital levels.

Table 24
Current capital and revised SA capital for SA banks

This table shows the weighted average capital requirements for categories (Cantonal and Other) of Swiss SA banks under the current SA rule and the revised credit risk SA rules proposed in BCBS 307 and BCBS 347. Figures are expressed in CHF Million.

Bank groups	**Banks**	**Corp. Financing**	**Comm. Mortgage**	**Res. Mortgage**	**Sum**
			Current capital		
Cantonal Banks	204	2,156	4,802	9,379	16,542
Other	1,061	2,169	2,845	12,582	18,603
All SA banks	1,266	4,326	7,646	21,907	35,145
			BCBS 307 revised SA capital		
Cantonal Banks	272	3,869	4,584	9,182	17,908
Other	1,819	4.043	2,641	11,926	20,429
All SA banks	2,091	7,912	7,226	21,109	38,337
% change between RSA and SA	65%	103%	-6%	-4%	12%
			BCBS 347 revised SA capital		
Cantonal Banks	207	3,908	4,355	10,618	19,088
Other	1,081	4,249	2,510	13,790	21,630
All SA banks	1,288	8,157	6,865	24,407	40,717
% change between RSA and SA	2%	89%	-10%	11%	16%

The introduction of the revised SA makes almost no direct difference to the IRB banks. (The only slight change evident in Corporate risk weights occurs because while predominantly applying the IRBA rules, these banks calculate capital for a small proportion of their Corporate exposures under SA rules.) The introduction of revised SA-capital-based floors has a very large impact, however, on the capital of the IRB banks.

Table 25
Weighted average risk weights for IRB banks under different scenarios

This table shows the weighted average risk weights of the three IRB banks we study, under existing rules and under the revised SA rules of BCBS 307. We present results under different assumptions about how IRBA risk weight floors would be linked to revised SA risk weights. Specifically, we suppose (i) that IRBA risk weight floors are set to different percentages (60%, 70% and 80%) of revised SA risk weights and (ii) that floors are imposed at individual exposure, asset class and overall bank level. We show results for exposures to banks, corporate loans, commercial and residential mortgages.

	BCBS 307 revised SA					
Bank groups	**Banks**	**Corp. Financing**	**Comm. Mortgage**	**Res. Mortgage**	**Spec. Lending**	**Wtd. Avg.**
Current risk weights	30%	43%	17%	11%	29%	19%
RSA without floor	30%	48%	17%	11%	29%	21%
RSA exposure level 60% floor	33%	78%	50%	24%	73%	43%
RSA asset class level 60% floor	30%	75%	50%	22%	72%	41%
RSA bank level 60% floor	21%	75%	50%	22%	72%	41%
RSA exposure level 70% floor	35%	88%	59%	27%	85%	49%
RSA asset class level 70% floor	30%	86%	59%	26%	85%	47%
RSA bank level 70% floor	24%	86%	59%	26%	85%	47%
RSA exposure level 80% floor	37%	98%	67%	30%	97%	55%
RSA asset class level 80% floor	31%	97%	67%	29%	97%	54%
RSA bank level 80% floor	27%	97%	67%	29%	97%	54%

	BCBS 347 revised SA					
Bank groups	**Banks**	**Corp. Financing**	**Comm. Mortgage**	**Res. Mortgage**	**Spec. Lending**	**Wtd. Avg.**
Current risk weights	30%	43%	17%	11%	29%	19%
RSA without floor	32%	43%	17%	11%	34%	23%
RSA exposure level 60% floor	38%	61%	50%	27%	46%	40%
RSA asset class level 60% floor	35%	53%	50%	26%	44%	37%
RSA bank level 60% floor	31%	53%	50%	26%	41%	37%
RSA exposure level 70% floor	41%	67%	58%	31%	51%	45%
RSA asset class level 70% floor	38%	61%	58%	30%	49%	43%
RSA bank level 70% floor	36%	61%	58%	30%	48%	43%
RSA exposure level 80% floor	45%	73%	66%	35%	57%	50%
RSA asset class level 80% floor	42%	69%	66%	35%	55%	49%
RSA bank level 80% floor	41%	69%	66%	35%	55%	49%

Under the BCBS 307 revised SA, the 60% floor imposed at the exposure level boosts IRB banks Corporate, Commercial Mortgage, Residential Mortgage and Specialized Lending risk weights from 43%, 17%, 11% and 29% to 79%, 50%, 24% and 73%, respectively. When an 80% floor is imposed at the exposure level, the risk weights for these four asset classes rise to 98%, 67%, 30% and 97%. These increases exceed factors of 2, 4, 2 and 3. Weighted average risk weights (across all IRB banks and the five asset classes we consider) go from 19% to 55%, a factor exceeding 2.

Under BCBS 347 revised SA, the 60% floor imposed at the exposure level boosts IRB banks Bank, Corporate, Commercial Mortgage , Residential Mortgage and Specialized Lending risk weights from 30%, 43%, 17%, 11% and 29% to 38%, 61%, 50% , 27% and 46%, respectively. When an 80% floor is imposed at the exposure level, the risk weights for these four asset classes rise to 45%, 73%, 66%, 35% and 57%. Weighted average risk weights go from 19% to 50%, a factor exceeding 2.

Figure 5. IRB bank RW changes with 80% asset class level floor

The figure shows percentage changes in weighted average IRBA bank risk weights for four exposure categories: Bank Exposures, Corporate Loans and Commercial and Residential Mortgages. The calculations are performed assuming an asset class level floor equal to 80% of the revised credit risk SA risk weights. All except bank exposure risk weights are substantially increased by the introduction of the revised credit SA risk weight floor.

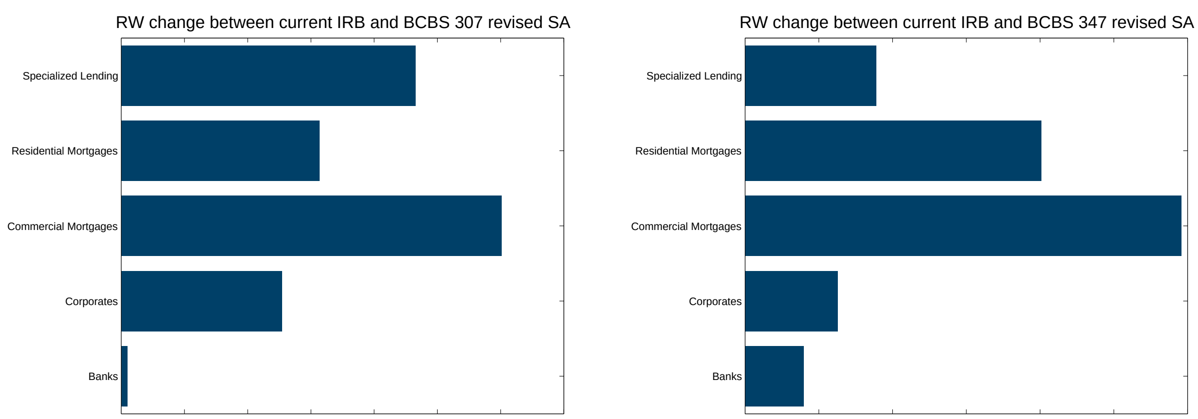


Table 26

Weighted average capital requirements for IRB banks

The upper panel show the capital requirements (in CHF millions) of the three Swiss IRB banks for individual asset classes under different scenarios. The lower panel shows the implied percentage changes in the three banks asset-class-specific capital compared to current capital levels. Total capital is doubled when an 80% floor is implemented. Under BCBS 307, for Corporate Loans and Commercial Mortgages, capital is 214% and 210% greater when an 80% asset class level floor is introduced. Under BCBS 347, for Bank, Corporate, and Commercial Mortgage exposures, capital is 208%, 279% and 207% greater when an 80% asset class level floor is introduced.

	Banks	Corp.	Cml. Mtg.	Res. Mtg.	S.L.	Total	Banks	Corp.	Cml. Mtg.	Res. Mtg.	S.L.	Total
	Weighted average capital requirements (CHFbn)											
Current capital	0.27	1.67	0.91	5.09	1.34	9.29	0.27	1.67	0.91	5.09	1.34	9.29
RSA without floor	0.31	2.30	0.89	4.95	1.36	9.80	0.54	3.17	0.85	4.74	1.66	10.96
RSA exposure level 60% floor	0.37	4.17	2.10	5.79	3.66	16.10	0.77	5.57	2.10	6.48	2.58	17.50
RSA asset class level 60% floor	0.34	3.93	2.06	5.52	3.58	15.43	0.71	4.86	2.10	6.13	2.47	16.26
RSA bank level 60% floor	0.27	3.92	2.06	5.50	3.57	15.32	0.62	4.86	2.10	6.13	2.30	16.01
RSA exposure level 70% floor	0.40	4.80	2.48	6.39	4.30	18.37	0.83	6.08	2.45	7.40	2.86	19.62
RSA asset class level 70% floor	0.35	4.65	2.47	6.11	4.28	17.87	0.77	5.60	2.45	7.15	2.76	18.73
RSA bank level 70% floor	0.28	4.65	2.47	6.11	4.28	17.79	0.73	5.60	2.45	7.15	2.69	18.61
RSA exposure level 80% floor	0.43	5.34	2.83	7.20	4.91	20.71	0.90	6.67	2.80	8.36	3.16	21.89
RSA asset class level 80% floor	0.36	5.25	2.83	6.98	4.90	20.32	0.84	6.34	2.80	8.17	3.07	21.22
RSA bank level 80% floor	0.31	5.25	2.83	6.98	4.90	20.27	0.83	6.34	2.80	8.17	3.07	21.21
	Change in capital (%)											
RSA exposure level 60% floor	37	150	130	14	174	73	182	233	131	27	93	88
RSA asset class level 60% floor	24	135	126	8	168	66	160	190	130	20	85	75
RSA bank level 60% floor	-2	134	125	8	167	65	128	190	130	20	72	72
RSA exposure level 70% floor	48	187	172	25	222	98	204	264	169	45	114	111
RSA asset class level 70% floor	28	178	171	20	220	92	184	235	168	40	106	102
RSA bank level 70% floor	2	178	171	20	220	92	166	235	168	40	101	100
RSA exposure level 80% floor	57	219	210	41	267	123	230	299	207	64	136	136
RSA asset class level 80% floor	32	214	210	37	266	119	208	279	207	60	130	128
RSA bank level 80% floor	16	214	210	37	266	118	204	279	207	60	1630	128

Note that an exposure level floor is more conservative in its impact on capital than an asset class level floor which, in turn, is more conservative than a bank level floor. This intuitive finding results from the fact that there may be offsets when the floor is applied at a more aggregate level. However, imposing capital floors at the three different levels leads to broadly similar results in practice. Figure 5 shows the overall risk weight impact by asset class with 80% asset class level floors. Under BCBS 307, risk weights rise by 5% for Bank exposures, 127% for Corporate exposures, 301% for Commercial Mortgages, 157% for Residential Mortgages and 233% for Specialized Lending. While under BCBS 347, risk weights rise by 40%, 63%, 296%, 201% and 89% for Banks, Corporates, Commercial Mortgages, Residential Mortgages and Specialized Lending respectively.

Table 27
Current and revised SA capital (with 80% asset class level floor) for all banks

The table shows current capital (broken down by asset class) for all banks and the capital implied by the revised credit risk SA and an 80% asset level floor for IRB banks. Under BCBS 307, total capital for Bank Exposure, Corporate Loans, Commercial and Residential Mortgages rises by 59%, 134%, 17% and 4%, respectively. Capital requirement figures are expressed in CHF million. While under BCBS 347, the figures are 38%, 142%, 13%, 21% and 130%.

Bank groups	**Banks**	**Corporates**	**Cml. Mortgage**	**Res. Mortgage**	**Specialized Lending**	**Total**
Current capital	1,538	5,998	8,559	27,002	1,337	44,433
			BCBS 307 revised SA			
Revised SA capital	2,451	14,032	10,054	28,090	4,897	59,523
Change in capital	59%	134%	17%	4%	266%	34%
			BCBS 347 revised SA			
Revised SA capital	2,128	14,498	9.662	32,581	3,069	61,938
Change in capital	38%	142%	13%	21%	130%	39%

Table 26 shows the impact on the capital of the IRB banks of the various scenarios so far considered. Overall (based on total capital across IRB banks and the five Swiss loan asset classes we consider), under BCBS 307, capital is 119% higher than current levels, if an asset class level 80% floor is introduced. The increases for Corporate and Commercial Mortgage exposures are 214% and 210%, while capital held against Residential Mortgages rise by just 37%. In monetary terms, the capital that the three IRB banks hold against their Swiss lending rises from CHF 9.29 billion to CHF 20.32 billion, in this case. Under BCBS 347, the total capital is 128% higher than current levels, if an asset class level 80% floor is applied. Capital held against Bank, Corporate, Commercial and Residential Mortgage exposures are 208%, 279%, 207% and 60% higher than the current levels.

Figure 6. Change in capital for all banks (with 80% asset class level floor)
The figure shows the percentage change in the total capital of Swiss banks, broken down by asset class, when the current rules are replaced with the revised credit risk SA and 80% asset-class level floors.

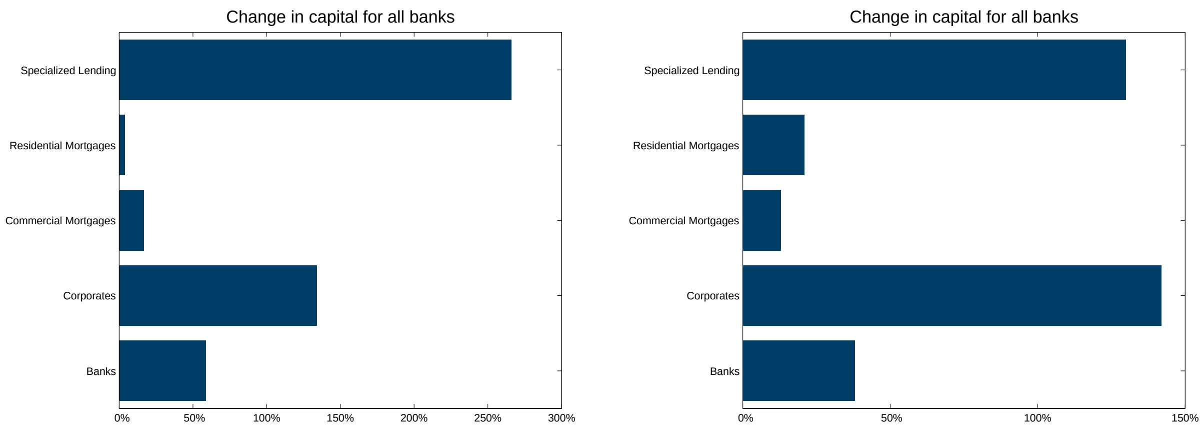


Table 27 shows the impact on the total capital that all banks hold against different asset classes. If an asset-class-level 80% floor is introduced for IRB banks, under BCBS 307, the increases in capital for exposures to Banks, Corporates and Specialized Lending are 59%, 134% and 266%. Capital held against Commercial Mortgages and Residential Mortgages rises by just 17% and 4%, in the same case. Under BCBS 347, the increase in capital held against Bank, Corporate, Commercial Mortgage, Residential Mortgage and Specialised Lending exposures are 38%, 142%, 13%, 21% and 130% respectively.

We now turn to the spread implications of the Basel Committees proposed BCBS 306, 307 and 347 capital rule changes. We calculate the spread impact using equations (4) and (4) in Section 4. We multiply post-rule change risk weights by the relevant capital target to obtain the per-Swiss-franc capital level under the new rules. We adjust for the leverage ratio target if the bank is a SIFI as in equation (4) and impose the relevant floor if this is included in the scenario we are examining. We multiply the resulting per Swiss-franc capital by the required equity return. We subtract off the pre-rule-change capital multiplied by a pre-rule-change required return on equity. Table 28 shows the resulting weighted average (across individual banks) spread impacts for SA banks, specifically for Cantonal and Other banks. We report spread impacts assuming that the capital rule changes reduce leverage and hence lead to a reduction in the cost of equity. The calculation of the reduction in cost of equity employs the Swiss bank regression 4 results (with both bank and year dummies) from Table 21.

Table 28
Spread impact in basis points for SA banks

The table shows the impact on the spreads charged by Swiss SA banks of replacing current rules with the revised credit risk SA. Units are basis points. The upper panel shows results when the capital impact is based on CET1 capital targets alone, while the lower panel shows results when the capital change is based on the Total Capital target ratio. The spread impacts are calculated assuming equity returns that adjust endogenously as total bank capital levels change. Spreads on commercial and residential mortgages fall slightly while those on corporate loans increase by 44 and 67 basis points (depending on the capital target ratio employed) when the BCBS 307 revised credit risk SA is introduced. Under BCBS 347, the spread impact on Corporate is similar to the figure from BCBS 307, but the spread impact on Residential Mortgages increases by 4 and 5 base points rather than decreasing when BCBS 307 is applied.

	Banks	**Corp.**	**Cmt. Mtg.**	**Res. Mtg.**	**Wtd. Avg.**	**Banks**	**Corp.**	**Cmt. Mtg.**	**Res. Mtg.**	**Wtd. Avg.**
						CET1 capital target				
Cant. Banks	6	41	-3	-1	4	0.1	33	-7	4	5
Other	8	47	-7	-2	4	0.0	42	-11	4	6
All SA banks	8	44	-5	-1	4	0.0	38	-9	4	6
						Total capital target				
Cant. Banks	8	60	-5	-1	6	0.1	49	-10	5	8
Other	13	72	-10	-3	6	0.1	64	-16	5	9
All SA banks	12	67	-7	-2	6	0.1	57	-12	5	9

The SA bank spread impacts shown in Table 28 are sizeable for exposures to Corporates under both BCBS 307 and BCBS 347. Under BCBS 347, the spread for Corporates rises by 38 basis points for all SA banks when a CET1 capital target is employed and by 57 basis points when a total capital target is used. SA bank spreads for Commercial Mortgages drop by 12 basis points with a CET1 target ratio when a total capital target ratio is employed under BCBS 347. Table 29 shows the spread impact of introducing the revised SA and an asset class level floor for weighted averages of IRB banks and IRB and SA banks combined for the different asset classes under assumptions of (i) CET1 and (ii) total capital target ratios and a reduced cost of equity through a Modigliani-Miller effect. Assuming (ii) rather than (i) boosts the impact substantially, as one might expect.

When the BCBS 307 rules are applied, applying a CET1 target ratio, one finds that IRB and SA banks Corporate spreads are 58 basis points higher with the 80% revised SA floor, while Commercial Mortgage, Residential Mortgage and Specialised Lending spreads are 14, 2 and 71 basis points higher. When the total capital target ratio is applied, the spread increases are 93, 23, 3 and 122 basis points. The weighted average across asset classes of spread impacts is 13 basis points for the CET1 target ratio and 21 basis points for the total capital target ratio.

When the BCBS 347 rules are applied, applying a CET1 target ratio, Bank, Corporate, Commercial Mortgages, Residential Mortgages and Specialised Lending spreads are 7, 63, 11, 7 and 41 basis points higher with the 80% revised SA floor. When the total capital target ratio is applied, the spread increases are 12, 103, 18, 11 and 71 basis points. The weighted average across asset classes of spread impacts is 15 basis points for the CET1 target ratio and 24 basis points for the total capital target ratio. Figures 7 and 8 show the spread effects graphically.

Table 29

Spread impact in basis points (asset class level floorfor IRB banks)

The table shows the weighted average impacts (in basis points) on the spreads charged by IRBA and SA Swiss banks of introducing the revised credit risk SA and 80%, asset-class-level IRBA capital floors. Results are exhibited assuming the capital change is based on the CET1 capital target ration or the Total Capital target ratio. Results are reported for Bank Exposures, Corporate Loans, Commercial and Residential Mortgages.

	BCBS 307 revised SA						BCBS 347 revised SA					
Wtd. Avg. of	**Banks**	**Corp.**	**Cml. Mtg.**	**Res. Mtg.**	**S.L.**	**Wtd. Avg.**	**Banks**	**Corp.**	**Cml. Mtg.**	**Res. Mtg.**	**S.L.**	**Wtd. Avg.**
CET1 capital target												
60% floor												
IRB Banks	7	52	35	3	47	20	45	73	35	5	31	21
IRB and SA Banks	8	47	8	0	47	9	5	51	5	4	31	10
70% floor												
IRB Banks	7	67	46	5	59	26	51	90	45	10	36	29
IRB and SA Banks	8	53	11	0	59	11	6	57	8	6	36	13
80% floor												
IRB Banks	8	81	56	9	71	33	57	107	55	16	41	36
IRB and SA Banks	8	58	14	2	71	13	7	63	11	7	41	15
Total capital target												
60% floor												
IRB Banks	13	90	56	4	80	323	77	126	56	8	53	36
IRB and SA Banks	12	75	13	0	80	14	9	82	9	6	53	17
70% floor												
IRB Banks	13	115	73	8	101	43	88	155	72	17	61	48
IRB and SA Banks	12	84	18	1	101	18	10	93	13	8	61	20
80% floor												
IRB Banks	14	139	89	15	122	55	98	184	88	25	71	60
IRB and SA Banks	12	93	23	3	122	21	12	103	18	11	71	24

Figure 7. Spread impact (in bps) for all banks (asset class level floor, CET1 target)
The figure shows spread impacts (in basis points and allowing for endogenous cost of equity) for all banks. The spread impacts are weighted by banks relative exposure volumes and assume the revised credit risk SA is introduced with asset-class-level 80% IRBA risk weight floor and that the capital impact is based on the CET1 ratio.

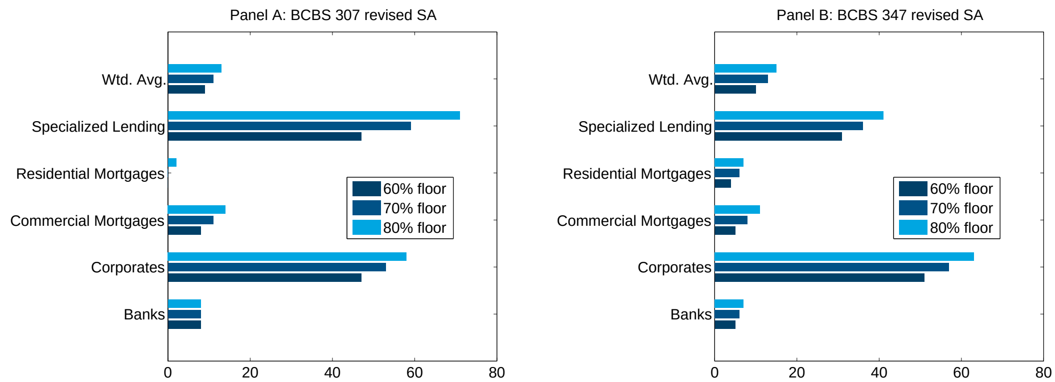


Figure 8. Spread impact (bps) across all banks (asset class level floor and total capital target)
The figure shows spread impacts (in basis points and allowing for endogenous cost of equity) for all banks. The spread impacts are weighted by banks relative exposure volumes and assume the revised credit risk SA is introduced with asset-class-level 80% IRBA risk weight floor and that the capital impact is based on the Total Capital.

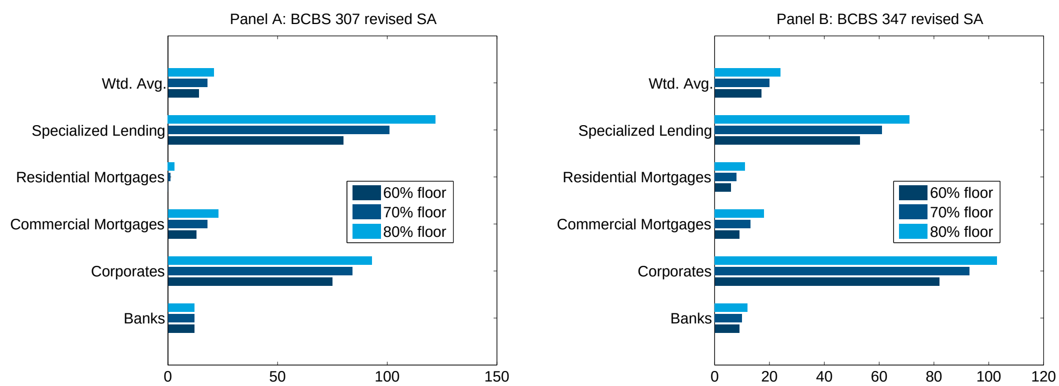


Table 30
Monetary impact per year

The table shows the annual cost in CHF millions of introducing the revised credit risk SA and 80% assetclass-level risk weights floors for IRB banks. The cost is calculated by multiplying individual bank spread impacts by their exposure volumes in the relevant asset class.

	BCBS 307 revised SA)						BCBS 347 revised SA					
Wtd. Avg. of	Banks	Corp.	Cml. Mtg.	Res. Mtg.	S.L.	Total	Banks	Corp.	Cml. Mtg.	Res. Mtg.	S.L.	Total
CET1 capital target												
RSA exposure level 60% floor	72	619	89	2	246	1,027	53	718	55	340	125	1,456
RSA asset class level 60% floor	69	596	88	-17	243	978	47	640	55	307	114	1,311
RSA bank level 60% floor	62	591	85	-23	239	953	38	640	55	307	98	1,285
RSA exposure level 70% floor	75	679	126	56	307	1,243	59	771	90	433	152	1,698
RSA asset class level 70% floor	69	664	126	28	306	1,193	53	718	90	409	143	1,596
RSA bank level 70% floor	62	664	126	27	306	1,185	48	719	90	409	136	1,583
RSA exposure level 80% floor	77	737	162	136	367	1,479	66	833	126	529	181	1,959
RSA asset class level 80% floor	70	727	162	114	367	1,440	60	797	126	511	173	1,881
RSA bank level 80% floor	66	727	162	114	367	1,436	59	797	126	511	173	1,880
Total capital target												
RSA exposure level 60% floor	112	987	148	16	423	1,686	91	1,168	100	505	213	2,344
RSA asset class level 60% floor	107	948	147	-14	417	1,604	81	1,035	100	451	195	2,101
RSA bank level 60% floor	94	939	143	-24	410	1,561	65	1,035	100	452	169	2,056
RSA exposure level 70% floor	117	1,091	207	103	527	2,045	101	1,259	157	653	260	2,742
RSA asset class level 70% floor	107	1,064	207	58	526	1,962	92	1,170	157	614	245	2,570
RSA bank level 70% floor	95	1,064	207	57	525	1,949	83	1,170	157	615	233	2,548
RSA exposure level 80% floor	121	1,189	265	232	631	2,437	114	1,366	213	807	311	3,172
RSA asset class level 80% floor	109	1,172	265	196	630	2,372	103	1,304	213	778	296	3,042
RSA bank level 80% floor	101	1,172	265	196	630	2,365	101	1,304	214	778	296	3,040

Table 31

PDV of monetary impact assuming a 3% discount rate

The table shows the present discounted cost in CHF millions of introducing the revised credit risk SA and 80% asset-class-level risk weights floors for IRB banks. The cost is calculated by assuming a perpetual annual cost as exhibited in Table 30 and discounting this by 3%.

	BCBS 307 revised SA)						BCBS 347 revised SA					
Wtd. Avg. of	Banks	Corp.	Cml. Mtg.	Res. Mtg.	S.L.	Total	Banks	Corp.	Cml. Mtg.	Res. Mtg.	S.L.	Total
CET1 capital target												
RSA exposure level 60% floor	2,409	20,626	2,953	52	8,199	34,240	1,767	23,919	1,824	11,344	4,151	43,005
RSA asset class level 60% floor	2,297	19,859	2,926	-579	8,086	32,589	1,559	21,326	1,827	10,222	3,799	38,734
RSA bank level 60% floor	2,054	19,689	2,842	-782	7,955	31,758	1,263	21,334	1,834	10,249	3,283	37,962
RSA exposure level 70% floor	2,497	22,643	4,195	1,855	10,243	41,433	1,963	25,695	3,009	14,432	5,061	50,159
RSA asset class level 70% floor	2,300	22,129	4,195	926	10,212	39,762	1,779	23,948	3,013	13,633	4,761	47,135
RSA bank level 70% floor	2,075	22,118	4,190	916	10,204	39,502	1,611	23,952	3,016	13,645	4,525	46,749
RSA exposure level 80% floor	2,581	24,555	5,388	4,539	12,249	49,312	2,203	27,764	4,195	17,645	6,044	57,851
RSA asset class level 80% floor	2,335	24,231	5,390	3,806	12,232	47,994	2,001	26,570	4,198	17,039	5,767	55,575
RSA bank level 80% floor	2,196	24,232	5,390	3,809	12,234	47,861	1,960	26,570	4,199	17,040	5,767	55,536
Total capital target												
RSA exposure level 60% floor	3,754	32,906	4,935	527	14,084	56,197	3,042	38,929	3,332	16,822	7,110	69,235
RSA asset class level 60% floor	3,554	31,594	4,896	-478	13,894	53,461	2,687	34,496	3,338	15,018	6,508	62,048
RSA bank level 60% floor	3,139	31,296	4,759	-810	13,665	52,048	2,169	34,508	3,349	15,062	5,630	60,718
RSA exposure level 70% floor	3,894	36,355	6,914	3,419	17,582	68,164	3,376	41,974	5,222	21,766	8,669	81,007
RSA asset class level 70% floor	3,558	35,474	6,914	1,927	17,530	65,404	3,059	38,988	5,230	20,482	8,160	75,920
RSA bank level 70% floor	3,174	35,454	6,906	1,910	17,516	64,960	2,765	38,993	5,235	20,502	7,756	75,251
RSA exposure level 80% floor	4,039	39,637	8,818	7,719	21,021	81,234	3,784	45,520	7,115	26,912	10,354	93,684
RSA asset class level 80% floor	3,617	39,081	8,821	6,544	20,994	79,056	3,434	43,479	7,121	25,940	9,881	89,854
RSA bank level 80% floor	3,380	39,082	8,822	6,548	20,997	78,830	3,361	43,479	7,121	25,942	9,882	89,785

Our results may be compared to those of recent studies that have examined the impact of capital rules changes on spreads in Swiss loan markets. Basten and Koch (2014) use panel data on mortgage offers to examine whether Swiss banks raised mortgage lending rates because of the introduction of the Counter-Cyclical Buffer increase in capital target rates. (In February 2013, the Swiss authorities activated a Counter Cyclical Buffer requiring banks to increase CET1 capital by an amount equal to 1% of their risk-weighted domestic Residential Mortgages by September 2013.) Basten and Koch find that, following the change, banks charged on average 17 to 18 basis points more while insurers charged on average 26 to 28 basis points more. The later finding suggests that banks are the marginal price setters and that insurers took the opportunity created by pressure on bank capital to raise their lending spreads significantly. Table 30 contains annual flow costs of lending and discounted sums of future costs. One may observe that the costs are between CHF 1.0 billion and CHF 2.5 billion if BCBS 307 rules are applied and the costs are between CHF 1.3 billion and CHF 3.2 billion if BCBS 347 rules are applied. Assuming a discount rate of 3%, we present estimates of the present discounted sum of future costs in Table 31. Overall, the present discounted cost of the rule changes is between CHF 46 billion and CHF 115 billion under BCBS 307, while the cost is between CHF 43 billion and CHF 106 billion under BCBS 347.

6 Conclusion

This paper examines the impacts on the Swiss loan market of the capital rule changes proposed in BCBS 306, 307 and 347. The rule changes include

a) the substitution of a risk-indicator-based, revised SA for the current SA (especially in the case of BCBS 307),

b) changes in the treatment of undrawn loan facilities (particularly important in the case of BCBS 347)

c) the imposition of capital floors for IRB capital based on a percentage of revised SA capital.

We study the effects of these changes on the risk weights and capital levels of 37 Swiss banks and banking groups including three IRB banks. We then examine how the capital changes are likely to affect the lending rates of these banks in different segments of the Swiss loan market, specifically lending to (i) other Swiss banks, (ii) Corporates (iii) Commercial Mortgage borrowers and (iv) Residential Mortgage borrowers. If implemented in Switzerland, we estimate that the proposed changes in capital rules contained in BCBS 347 would increase capital for IRB banks for Bank, Corporate, Commercial and Residential Mortgage and Specialised Lending by 208%, 279%, 207%, 60% and 109%.

Assuming full "pass through" to borrowers, a weighted average of lending rates on Corporate loans for all IRB and SA banks would rise by between 63 and 103 basis points.An incomplete 50% pass through would lead to rises in lending rates of between 26 and 52 basis points.[27]

We calculate monetary impacts of the spread changes on the Swiss economy by multiplying weighted averages (across banks) of the spread changes with the volumes of outstanding loans and an assumed pass through parameter of 100%.[28] The resulting estimates suggest that the annual cost of the policy change would be between CHF 1.9 billion and CHF 3.0 billion while the total present discounted cost would be between CHF 62.7 billion and CHF 101.4 billion (assuming a 3

[27] We do not try to infer a pass through fraction for spreads changes consequent on changes in capital rules since (i) inferring such a pass through percentage is difficult and arbitrary and (ii) even if not passed through, spread changes impose costs on bank shareholders. Illustrating the difficulty of inferring pass through percentages, Cecchin (2011) looks at the pass though of bank funding costs (due to changes in market interest rates) to floating and fixed rate Swiss mortgage lending rates. The results are complex suggesting different degrees of competition in the fixed and floating rate segments of the market and consequent upward and downward inflexibility.

[28] We think it appropriate to perform these calculations assuming a 100% pass through as this gives a measure of the total cost on both borrowers and bank shareholders.

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7 Appendix

This section describes how we estimate the distribution of risk drivers for exposures to banks.

7.1 Assumptions

- We classify the Swiss banks into 3 groups: Large Banks, Cantonal Banks and Other Banks. For each bank group, we assume the credit exposures to the three bank groups are distributed as Table 32. Large banks' exposure is partially data driven and the rest is expert based.
- Table 34 shows the risk drivers (Net NPA (NNPA) ratios and CET1 ratios) for 48 Swiss banks. The risk drivers represented here are proxies rather than exact figures. These do not exactly match either the definition of CET1, or the definition of Net NPA ratio, as defined in the revisedSA approach.The following assumptions are made to derive the required ratios:
  1. Basel II Tier One Regulatory Capital ratio as proxy for CET1
  2. Modified definition of NNPA ratio, namely (Non-Performing Loans - Loan Loss Reserve)/(Total Earning Assets - Total Securities)
  3. Risk driver values taken from the 2013 End of year Financial statements

Table 32
Interbank credit risk exposure distribution for different bank groups

This table shows the assumptions we make regarding the exposure shares that each individual bank (within one of the three groups of banks) has with respect to other Swiss banks in the three different categories we consider. Hence, we suppose that, for each of the two large Swiss banks, 30% of its reported exposure to Swiss banks is with respect to the other large bank and 30% is with respect to cantonal banks. The assumed percentages were provided to us by a banker closely familiar with the Swiss interbank market and are based on the judgments of that individual.

	Large banks	**Cantonal Banks**	**Other banks**
Large Banks	30%	30%	40%
Cantonal Banks	40%	30%	30%
Other banks	50%	30%	20%

Each row represents the credit risk exposure distribution for that bank group. The number of banks in each bank group is given in Table 33.

Table 33
Numbers of banks by group

The Table shows the numbers of banks in each of the three categories we study, Large, Cantonal and Other Banks.

	Large banks	**Cantonal Banks**	**Other banks**
Count	2	14	32

Table 34
NNPA and CET1 ratios

The table shows the classification of a set of Swiss banks according to Net Non-Performing Asset and CET1 ratios and according to whether they are Large Banks, Cantonal Banks or Other Banks.

Bank names	**Bank group**	**Classification with respect to Net NPA proxy**	**Classification with respect to CET1 proxy**
Caisse d'Epargne d'Aubonne	Large banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Credit Suisse	Large banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
UBS	Large banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Reiffeisen	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Banca dello Stato del Cantone Ticino	Cantonal banks	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Banque Cantonale du Jura	Cantonal banks	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Banque Cantonale Vaudoise	Cantonal banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Basellandschaftliche Kantonalbank	Cantonal banks	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Basler Kantonalbank	Cantonal banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Berner Kantonalbank	Cantonal banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Luzerner Kantonalbank	Cantonal banks	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Schaffauser Kantonalbank	Cantonal banks	Net NPA $>$ 3%	12% $\leq$ CET1
St. Galler Kantonalbank	Cantonal banks	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Thurgauer Kantonalbank	Cantonal banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Walliser Kantonalbank	Cantonal banks	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Zuger Kantonalbank	Cantonal banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Zurcher Kantonalbank	Cantonal banks	Net NPA $\leq$ 1%	12% $\leq$ CET1
Bank J. Safra Sarasin	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Bank Linth	Other	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Clientis	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Coop Bank	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Julius Baer	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Migros Bank	Other	1% $<$ Net NPA $\leq$ 3%	7% $\leq$ CET1 $<$ 9.5%
Neue Aargauer Bank	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Bank CIC (Schweiz) AG	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Bernerland Bank	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Leumi Private Bank Ltd	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
BSI SA	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Coutts & Co Ltd	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
EFG International	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Freie Gemeinschaftsbank BCL	Other	Net NPA $>$ 3%	9.5% $\leq$ CET1 $<$ 12%
Bank Hapoalim (Switzerland)	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Privatbank IHAG Zurich	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Maerki Baumann & Co. AG	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Notenstein Private Bank Ltd	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Bank Morgan Stanley AG	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Valartis Group AG	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
BNP Paribas (Suisse) SA	Other	1% $<$ Net NPA $\leq$ 3%	7% $\leq$ CET1 $<$ 9.5%
Piguet Galland & Cie SA	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Pand Privatbank AG	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Regiobank Solothurn	Other	1% $<$ Net NPA $\leq$ 3%	12% $\leq$ CET1
Baloise Bank SoBa	Other	Net NPA $\leq$ 1%	9.5% $\leq$ CET1 $<$ 12%
Swissquote Group Holding Ltd.	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Acrevis Bank AG	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Union Bancaire Privee	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Banca Zarattini & Co SA	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1
Vontobel Group	Other	Net NPA $\leq$ 1%	12% $\leq$ CET1

7.2 Estimate distributions

Given the interbank credit exposure distribution (Table 32) for each bank group and the risk drivers (Table 34), we can estimate the risk driver distributions for each bank group in a simplified approach. The estimation steps are given as following:

- Step 1: We classify each bank into the three bank groups.
- Step 2: For each bank, determine which CET1 and NNPA bucket it belongs to given its

CET1 ratio and NNPA ratio.

- Step 3: For $g = 1 : 3$ (for each bank group), for $j = 1 : 3$ (for each bank group) and for $i = 1 : 48$ (for each bank) Calculate the i^{th} bank's weight as $w_i = w_{g,j}/N_j$ if bank i belongs to bank group j, where $w_{g,j}$,?is the total weight of bank group j as shown in row g in Table 32, N_j is the total number of banks in group j.
- Step 4: Calculate the probability for CET1 and NNPA bucket k as: $p_k = \sum_{i=1}^{N} w_i$, where $w_i = w_{g,j}/N_j$ if bank i belongs to CET1 and NNPA bucket k, otherwise $w_i = 0$. End

The estimated distribution is given in Table 35.

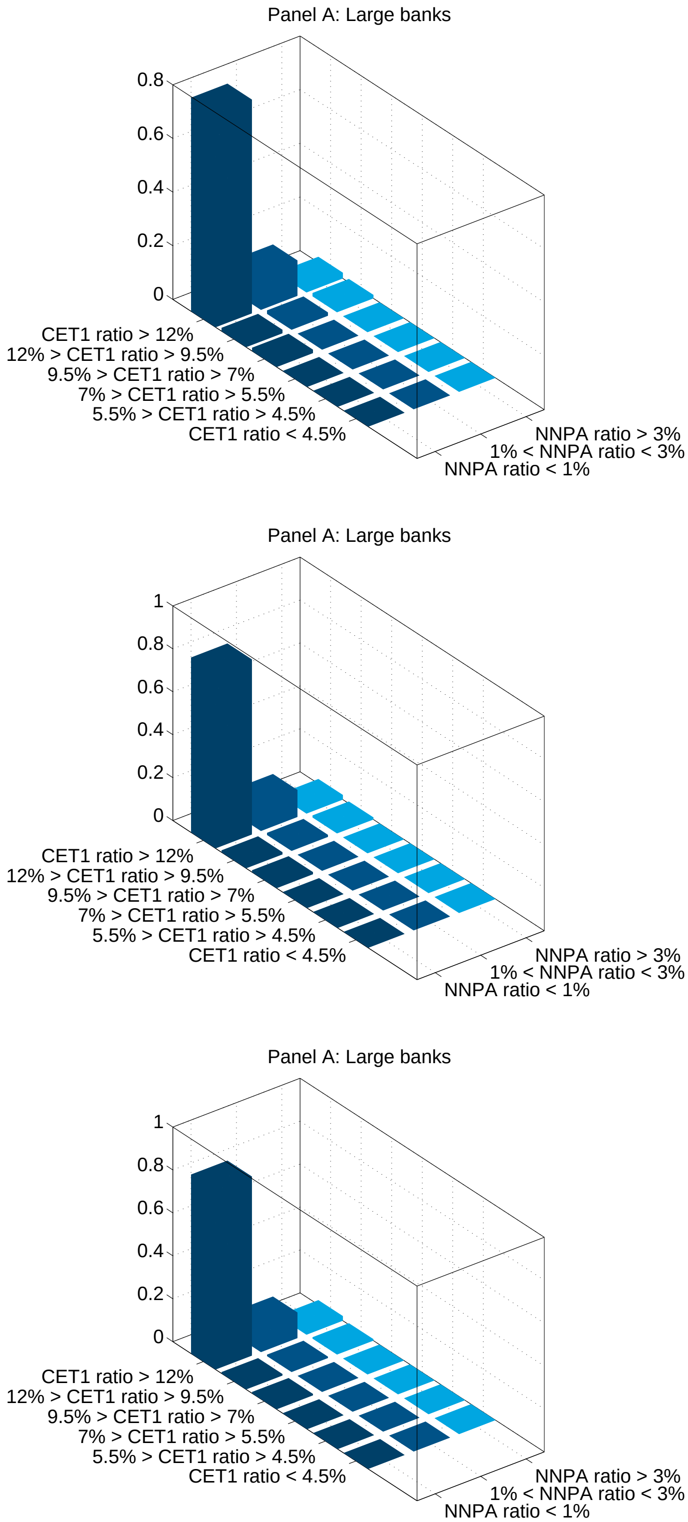
Table 35
Generated joint distribution of CET1 and NNPA

The table shows for individual banks in each of our three categories of banks the distributions (by Net Non-Performing Asset (NNPA) and Common Equity Tier 1 (CET1) ratios) of that bank's exposures to other Swiss banks in the three categories.

	CET1 ratio ≥ 12%	**12% > CET1 ratio ≥ 9.5%**	**9.5% > CET1 ratio ≥ 7%**	**7% > CET1 ratio ≥ 5.5%**	**5.5% > CET1 ratio ≥ 4.5%**	**CET1 ratio ≥ 4.5%**
			Large banks			
NNPA ratio ≤ 1%	79.64%	1.25%	1.25%	0.00%	0.00%	0.00%
1% < NNPA ratio ≤ 3%	13.21%	1.25%	0.00%	0.00%	0.00%	0.00%
NNPA ratio > 3%	2.14%	1.25%	0.00%	0.00%	0.00%	0.00%
			Cantonal banks			
NNPA ratio ≤ 1%	81.52%	0.94%	0.94%	0.00%	0.00%	0.00%
1% < NNPA ratio ≤ 3%	12.59%	0.94%	0.00%	0.00%	0.00%	0.00%
NNPA ratio > 3%	2.14%	0.94%	0.00%	0.00%	0.00%	0.00%
			Other banks			
NNPA ratio ≤ 1%	83.39%	0.63%	0.63%	0.00%	0.00%	0.00%
1% < NNPA ratio ≤ 3%	11.96%	0.63%	0.00%	0.00%	0.00%	0.00%
NNPA ratio > 3%	2.14%	0.63%	0.00%	0.00%	0.00%	0.00%

Figure 9. Generated joint distribution of CET1 and NNPA

The figure shows graphically the distributions contained in Table 35.



7.3 Distribution of unrated bank exposure

Table 36
Interbank credit risk exposure distribution for different bank groups

The table shows the interbank credit risk exposure distribution for different bank groups.

	Large banks	**Cantonal Banks**	**Other banks**
Large banks	0.3	0.4	0.3
Cantonal banks	0.3	0.3	0.3
Other banks	0.5	0.15	0.35

Based on the above assumptions we van estimate the distributions of unrated bank exposures as shown in Table 37.

Table 37
Distribution of unrated bank exposures

The table shows the distribution of unrated bank exposures.

	Grade A	**Grade B**	**Grade C**	**Total unrated**
Large banks	30.5%	4.5%	0.0%	35.0%
Cantonal banks	25.5%	4.5%	0.0%	30.0%
Other banks	19.8%	5.3%	0.0%	25.0%

Part III

Maximum Diversification Strategies Along Commodity Risk Factors

Simone Bernardi, Markus Leippold, and Harald Lohre

Abstract

Pursuing risk-based allocation across a universe of commodity assets, we find diversified risk parity (DRP) strategies to provide convincing results. DRP strives for maximum diversification along uncorrelated risk sources. A straightforward way to derive uncorrelated risk sources relies on principal components analysis (PCA). While the ensuing statistical factors can be associated with commodity sector bets, the corresponding DRP strategy entails excessive turnover because of the instability of the PCA factors. We suggest an alternative implementation of a DRP strategy that builds on a more stable anchor and implicitly allows for a uniform exposure to commodity risk premia.

Keywords: Commodity Strategies, Risk-Based Portfolio Construction, Risk Parity, Diversification

JEL Classification: G11; D81

1 Introduction

Commodity investing is often suggested for diversifying traditional stock-bond portfolios — for example, there is empirical evidence of negative correlation between stocks and commodities during stock market downturns, which makes commodities a perfect hedging instrument, see Bodie and Rosansky (2000). However, while there is plenty of evidence that adding commodities to an existing stock-bond portfolio can enhance its risk-return profile, there is less research on the diversification potential inherent within the universe of commodity assets.[1]

From a pure return perspective, Erb and Harvey (2006) document that the average annual excess return of individual commodity futures has historically been approximately zero. Hence, static long-only investments in commodities may not be necessarily profitable. In addition, the inherent heterogeneity within this asset class, paired with high volatility and excess kurtosis, very often offsets any positive average return. On the other side, the same authors document abnormal returns for specific combinations of commodities, which exhibit a forward curve with attractive term structure characteristics. To derive profitable commodity trading strategies, one should thus resort to momentum or commodity-term structure signals, see Fuertes, Miffre, and Rallis (2010). Recently, Fuertes, Fernandez-Perez, and Miffre (2016) document abnormal returns when trading long low-idiosyncratic volatility positions versus the high ones, thus evidencing an inverse risk-return relationship as prevailing in equities.[2]

We contribute to the literature by devising optimally diversified commodity portfolios along these commodity risk factors. As evidenced by Erb and Harvey (2006), diversification is key in generating performance in commodities. The standard approach to exploiting diversification benefits is to follow the classic mean-variance approach of Markowitz (1952) to optimally trade-off assets risk and return. Yet, despite the heterogeneity of commodity markets and the low pair-wise correlations across different commodity sectors, the ensuing portfolio construction will most likely be confounded by the estimation risk, especially the one for estimating expected returns. More recently, in pursuit of better diversified portfolios, alternative risk-based allocation techniques have

[1]See among others Kat and Oomen (2007) for an overview. Also, see Miffre (2016) for a recent survey for the literature on long-short commodity investing.

[2]See also Ang, Hodrick, Xing, and Zhang (2006).

become popular. Qian (2006, 2011) and Maillard, Roncalli, and Teiletche (2010) advocate the risk parity approach that allocates capital such that all assets contribute equally to portfolio risk.

The common rationale of all the above approaches is diversification. Still, diversification is a rather elusive concept, which is hardly made explicit in portfolio optimization studies. A notable exception, however, is Meucci (2009). Striving for diversification, he pursues principal component analysis (PCA) to extract uncorrelated risk sources inherent in the underlying assets. The resulting eigenvectors represent linear combinations of the underlying assets and are thus commonly referred to as principal portfolios.[3] For a portfolio to be well-diversified, its overall risk should be evenly distributed across these principal portfolios.

Recently, Lohre, Opfer, and Ország (2014) have adopted the framework of Meucci (2009) to determine maximum diversification portfolios in a multi-asset allocation study. Their strategy coincides with a risk parity strategy that allocates risk by principal portfolios rather than by the underlying assets. The authors demonstrate this diversified risk parity (DRP) strategy to provide convincing risk-adjusted performance, together with superior diversification properties, when benchmarked against other competing risk-based investment strategies. We explore a different route. In particular, we seek to exploit the diversification potential inherent in the commodity market by investing in two distinct uncorrelated decompositions of it: the principal portfolios (PPs) arising out of the PCA and minimum torsions (MTs) derived from the minimum rotation of a given commodity factor model. In an empirical study we benchmark these strategies against a set of well known diversification strategies under long-only constraints.

We find that a long-only DRP strategy that diversifies along the most relevant principal portfolios of the Standard and Poor's Goldman Sachs Commodity Index (GSCI) universe indeed delivers superior risk-adjusted performance in a 30-year backtest. The DRP strategy differs from the prevailing risk-based allocation schemes like $1/N$, minimum-variance, or traditional risk parity, since it is characterized by concentrated allocations that are altered actively whenever a significant change in risk structure calls for adjusting the risk exposure. As a result, when budgeting risk along principal portfolios the strategy entails a significant amount of turnover potentially eroding its added

[3]Partovi and Caputo (2004) have coined the term principal portfolios when recasting the efficient frontier in terms of these principal portfolios.

value in terms of return. Obviously, the amount of turnover is related to the instability of the principal portfolios. Moreover, PCA factors often lack a sound economic interpretation, which is complicating the decision to buy or sell a given principal portfolio. Risk-wise, one is indifferent to buying or selling a given principal portfolio.

To alleviate the above issues, we build on Meucci, Santangelo, and Deguest (2014) to come up with a more meaningful set of orthogonal factors. In particular, we consider an orthogonalized version of well known commodity risk factors. These factors are chosen in such a manner that they have minimum tracking error with regard to the original ones. Therefore, these orthogonalized factors are labeled minimum risk factor torsions. In this way, the risk model guiding the DRP strategy is anchored in more robust risk factors that implicitly determine the direction of trade. We document the associated DRP strategy to provide a comparable risk-return profile as the PCA version, but at a considerably lower turnover.

In addition, analyzing the risk structure of the competing alternatives, we find that the traditional risk parity strategy is similar to the $1/N$ strategy or the market indices in having a concentrated risk exposure. When it comes to diversification of weights, minimum-variance strategies typically prove to be rather concentrated in low-volatility assets. In the equity domain, this observation resonates with the finding of Scherer (2011) that minimum-variance strategies implicitly capture risk-based pricing anomalies inherent in the cross-section of stock returns, especially the low-volatility and low-beta anomalies. In this vein, a commodity factor model accounting for common risk factors is a prerequisite for explaining a given strategy's return. Moreover, we demonstrate that the performance of diversified risk parity strategies is derived from a uniform exposure to various risk factors.

Our paper is structured as follows. Section 2 describes the methodology of the risk-based asset allocation techniques. In Section 3, we foster intuition of the principal portfolios relative to the minimum risk factor torsions. Section 4 studies the empirical implementation of long-only risk-based strategies in the classic commodities universe. Section 5 concludes.

2 Managing Diversification

While there are many different ways to achieve diversification, we focus on two approaches, diversifying by principal portfolios and diversifying by minimum risk factor torsions. Below, we present their theoretical underpinnings and we introduce the benchmarking strategies that we use later in our empirical study.

2.1 Diversifying by principal portfolios

We consider a portfolio comprising N commodities with weight and return vectors $\mathbf{w}$ and $\mathbf{R}$, providing a portfolio return of $R_w = \mathbf{w'R}$. At the heart of diversification is the search for low-correlated assets. Although commodities are a heterogeneous asset class, the corresponding correlation figures will hardly be zero. Still, it is possible to construct uncorrelated assets from a given covariance matrix. Along these lines, Meucci (2009) extracts uncorrelated risk sources by applying a principal component analysis (PCA) to the covariance matrix Σ of the portfolio assets, i.e.,

$$\mathbf{\Sigma} = \mathbf{E\Lambda E}', \tag{1}$$

where $\mathbf{\Lambda} = \text{diag}(\lambda_1, ..., \lambda_N)$ is a diagonal matrix comprising $\mathbf{\Sigma}$'s eigenvalues that are assembled in descending order, $\lambda_1 \geq ... \geq \lambda_N$. The columns of matrix $\mathbf{E}$ represent the eigenvectors of $\mathbf{\Sigma}$ that define a set of N uncorrelated principal portfolios with variance λ_i for $i = 1, ..., N$ and returns $\tilde{R} = \mathbf{E}^{'}\mathbf{R}$. Hence, we can think of a given portfolio either in terms of its weights $\mathbf{w}$ in the original assets or in terms of its weights $\tilde{\mathbf{w}} = \mathbf{E}^{'}\mathbf{w}$ in the principal portfolios. Because principal portfolios are uncorrelated by design, the total portfolio variance is derived by simply computing a weighted average over their corresponding variances λ_i using weights $\tilde{w}_i^2$:

$$Var(R_w) = \sum_{i=1}^{N} \tilde{w}_i^2 \lambda_i. \tag{2}$$

Normalizing the principal portfolios' contributions by the portfolio variance then yields the diversification distribution:

$$p_i = \frac{\tilde{w}_i^2 \lambda_i}{Var(R_w)}, \qquad i = 1, ..., N. \tag{3}$$

Note that the diversification distribution is always positive, and that all p_i sum up to one.

Building on the above concept, Meucci (2009) conceives a portfolio to be well diversified when the principal portfolios contributions p_i are "approximately equal and the diversification distribution is close to uniform." Conversely, portfolios mainly loading on a single PP display a peaked diversification distribution. To aggregate the diversification distribution, Meucci (2009) chooses the exponential of its entropy for evaluating a portfolios degree of diversification:

$$\mathcal{N}_{Ent} = \exp\left(-\sum_{i=1}^{N} p_i \ln p_i\right). \tag{4}$$

Intuitively, we can interpret $\mathcal{N}_{Ent}$ as the number of uncorrelated bets. For instance, a completely concentrated portfolio is characterized by $p_i = 1$ for one i and $p_j = 0$ for $i \neq j$ resulting in an entropy of 0, which implies $\mathcal{N}_{Ent} = 1$. Conversely, $\mathcal{N}_{Ent} = N$ is obtained for a portfolio that is completely homogenous in terms of uncorrelated risk sources. In this case, $p_i = p_j = 1/N$ holds for all i, j, implying an entropy equal to $\ln(N)$ and $\mathcal{N}_{Ent} = N$.

Lohre, Opfer, and Ország (2014) implement the above-mentioned definition of a well-diversified portfolio by constructing an allocation strategy, which allocates equal risk budgets to every uncorrelated principal portfolio. We obtain the weights $\mathbf{w}_{DRP}^{PP}$ of this strategy by solving

$$\mathbf{w}_{DRP}^{PP} = \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \, \mathcal{N}_{Ent}(w), \tag{5}$$

where the weights $\mathbf{w}$ may possibly be restricted according to a set of constraints $\mathcal{C}$. Obviously, an inverse volatility strategy along the PPs is a feasible, but not unique, solution of (5). In fact, buying or selling a certain amount of a given principal portfolio gives rise to the same ex ante risk exposure. For K principal portfolios, there exist 2^K solutions, all of which are inverse volatility strategies reflecting all possible variations of long and short principal portfolios. Typically, most of these portfolios tend to be difficult to implement in practice because of rather infeasible portfolio weights. For the traditional risk parity strategy, Maillard, Roncalli, and Teiletche (2010) show that imposing positive asset weights guarantees a unique solution. Unfortunately, the positivity of asset weights is not a sufficient condition to determine a unique DRP strategy.

In that regard, Bruder and Roncalli (2012) and Roncalli and Weisang (2012) investigate general risk budgeting strategies and demonstrate that uniqueness is obtained when imposing constraints to the underlying risk factors. In our case, this requirement translates into imposing sign constraints to the principal portfolios. In doing so, we express a view with respect to the risk premium of each principal portfolio. While one could resort to elaborate forecasting techniques to derive these views, we pursue a more pragmatic approach. We equalize the desired sign of the principal portfolios with the sign of its corresponding historical risk premium. Thus, we intend to design a strategy that is geared towards exploiting long-term risk premia. The respective historical risk premia result from multiplying the current principal portfolios' weights with historical asset returns using an expanding time window.

2.2 Diversifying by minimum torsions

A principal component analysis provides just one possible orthogonal decomposition of the covariance matrix $\mathbf{\Sigma}$. While the principal portfolios are designed to capture most of the assets' variations, they are often perceived as being ad-hoc statistical factors that lack an economic interpretation and are rather unstable over time. Addressing these objections, Meucci, Santangelo, and Deguest (2014) suggest resorting to a factor model that can be orthogonalized in a way that ensures a minimum tracking error to the original factors. The authors start from a K-factor model $\mathbf{F} = (F_k)_{k=1}^{K}$ to explain asset returns and propose a methodology to change the standard representation of portfolio returns R_w,

$$R_w = \mathbf{w}'\mathbf{R} = \mathbf{b}'\mathbf{F}, \tag{6}$$

into a representation in terms of uncorrelated factors $\mathbf{F}_T$, i.e.,

$$R_w = \mathbf{b}'\mathbf{F} = \mathbf{b}_T'\mathbf{F}_T, \tag{7}$$

where $\mathbf{b}$ and $\mathbf{b}_T$ denote the loadings of the portfolio returns with respect to the factor model $\mathbf{F}$ and $\mathbf{F}_T$, respectively. Following Section 2.1 and using $\mathbf{R} = \mathbf{B}'\mathbf{F}$, where $\mathbf{B} \in \mathbb{R}^{K \times N}$, we can decompose the covariance matrix $\mathbf{\Sigma}$ as follows:

$$\mathbf{\Sigma} = \mathbf{B}'\mathbf{\Sigma}_F\mathbf{B} + \mathbf{u}, \tag{8}$$

where $\mathbf{u}$ captures assets' idiosyncratic risk that cannot be explained by the chosen factor structure. The next step is to construct an orthogonal decomposition of $\mathbf{F}$ by means of a linear transformation $\mathbf{F}_T = \mathbf{tF}$ where $\mathbf{t}$ is a suitable $K \times K$ rotation, or torsion, matrix.

The contribution of Meucci, Santangelo, and Deguest (2014) is to define a systematic way of constructing uncorrelated factor model representations by looking at the set of uncorrelated risk sources, which closely mimics the original factor model $\mathbf{F}$. In particular, the authors compute the uncorrelated factor model that represents the minimum torsion linear transformation of the original factor model.[4] As a consequence, the orthogonalized factors keep the original factor interpretation as close as possible. Among all linear transformations $\mathbf{t}$, which ensure factors to be uncorrelated, they select the minimum torsion $\mathbf{t}_{MT}$ that minimizes the distance to the original factors,

$$\mathbf{t}_{MT} = \underset{Corr(\mathbf{tF})=\mathbf{I}_{K\times K}}{\operatorname{argmin}} \sqrt{\frac{1}{K}\sum_{k=1}^{K} Var\left(\frac{\mathbf{t}'\mathbf{F}_k - \mathbf{F}_k}{\sigma_k^F}\right)}, \tag{9}$$

where σ_k^F denotes the volatility of the factor $\mathbf{F}_k$. Under this minimum torsion $\mathbf{t}_{MT}$, the systematic risk of a given portfolio can be decomposed as follows:

$$\mathbf{B}'\mathbf{\Sigma}_F\mathbf{B} = \mathbf{B}'\mathbf{t}_{MT}^{-1}\mathbf{\Sigma}_{MT}\mathbf{t'}_{MT}^{-1}\mathbf{B}, \tag{10}$$

where $\mathbf{\Sigma}_{MT} = \mathrm{diag}(\sigma^2_{MT,1}, ..., \sigma^2_{MT,K})$ is a diagonal matrix comprising minimum risk factor torsion variances. Analogous to the PCA decomposition, we can write the portfolio in terms of physical weights $\mathbf{w}$ in the original assets, or in terms of torsion weights $\mathbf{w}_{MT}$ in the minimum torsion risk factors $\mathbf{F}_{MT} = \mathbf{t}_{MT}\mathbf{F}$. It holds that $\mathbf{w}_{MT} = \mathbf{t'}_{MT}^{-1}\mathbf{Bw}$ and the minimum risk factor torsions have returns $\mathbf{R}_{MT} = \mathbf{t'}_{MT}^{-1}\mathbf{BR}$. Again, the total portfolio variance can simply be computed as a weighted

[4]See http://symmys.com/node/599 for the corresponding Matlab code.

average over the uncorrelated factors variances $\sigma^2_{MT,k}$ using minimum risk factor torsion weights $w_{MT,k}$:

$$Var(R_w) = \sum_{k=1}^{K} w^2_{MT,k}\sigma^2_{MT,k}. \tag{11}$$

The diversification distribution (3) and the number of uncorrelated bets in (4) follow using

$$p_{MT,k} = \frac{w^2_{MT,k}\sigma^2_{MT,k}}{Var(R_w)}, \qquad k = 1, ..., K. \tag{12}$$

Analogous to optimization (5), we obtain the weights $\mathbf{w}^{MT}_{DRP}$ by maximizing the corresponding entropy measure

$$\mathbf{w}^{MT}_{DRP} = \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \exp\left(-\sum_{k=1}^{K} p_{MT,k} \ln p_{MT,k}\right). \tag{13}$$

In contrast to the diversified risk parity strategy (based on principal portfolios), the diversified risk parity strategy along minimum risk factor torsions naturally entertains a view on the sign of risk factors, which is consistent with the underlying economic factor model. Hence, one does not need to impose further restrictions to render the optimal strategy unique.

2.3 Benchmark strategies

For benchmarking the diversified and principal risk parity strategy, we consider four alternative risk-based asset allocation strategies: $1/N$, minimum-variance, risk parity, and the most diversified portfolio of Choueifaty and Coignard (2008). First, we implement the $1/N$ strategy that monthly rebalances to an equally weighted allocation scheme. Hence, the portfolio weights $\mathbf{w}_{1/N}$ are simply:

$$\mathbf{w}_{1/N} = \frac{\mathbf{1}}{N}. \tag{14}$$

Second, we compute the minimum-variance (MV) portfolio by building on an expanding time window, starting from 36 monthly observations, for the covariance-matrix estimation. We derive the corresponding weights $\mathbf{w}_{MV}$ from

$$\mathbf{w}_{MV} = \underset{\mathbf{w}}{\operatorname{argmin}}\, \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}, \tag{15}$$

subject to the full investment and positivity constraints $\mathbf{w}'\mathbf{1} = 1$ and $\mathbf{w} \geq \mathbf{0}$.

Third, we construct the original risk parity (RP) strategy by allocating capital in such a manner that the asset classes risk budgets contribute equally to the overall portfolio risk. Note that these risk budgets also depend on an expanding time-window estimation, starting from 36 monthly returns. Since there are no closed-form solutions available, we follow Maillard, Roncalli, and Teiletche (2010) to obtain $\mathbf{w}_{RP}$ numerically via

$$\mathbf{w}_{RP} = \underset{\mathbf{w}}{\operatorname{argmin}} \sum_{i=1}^{N}\sum_{j=1}^{N} \left(w_i(\boldsymbol{\Sigma}\mathbf{w})_i - w_j(\boldsymbol{\Sigma}\mathbf{w})_j\right)^2 \tag{16}$$

which essentially minimizes the variance of the risk contributions. Again, the above-mentioned full investment and positivity constraints apply.

Fourth, we use the approach of Choueifaty and Coignard (2008) to build maximum diversification portfolios. To this end, the authors define a portfolio diversification ratio $D(\mathbf{w})$:

$$D(\mathbf{w}) = \frac{\mathbf{w}' \cdot \boldsymbol{\sigma}}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}}, \tag{17}$$

where $\boldsymbol{\sigma}$ is the vector of portfolio assets volatility. Thus, their most-diversified portfolio (MDP) simply maximizes the ratio of two distinct definitions of portfolio volatility — i.e., the ratio of the average portfolio assets volatility and the total portfolio volatility. We obtain MDP's weights vector $\mathbf{w}_{MDP}$ by numerically computing:

$$\mathbf{w}_{MDP} = \underset{\mathbf{w}}{\operatorname{argmax}}\, D(\mathbf{w}). \tag{18}$$

As for the other benchmarking strategies, we enforce the full investment and positivity constraints.

3 Constructing Uncorrelated Commodity Risk Factors

Before analyzing principal portfolios and minimum risk factor torsions, we start with a brief discussion of our data and provide some descriptive statistics.

3.1 Data and preliminary analysis

We investigate risk-based commodity strategies using the 24 commodities included in a recent version of the S&P Goldman Sachs Commodity Index (GSCI) from January 1983 to December 2014.[5] Exposures to commodities are established by trading the corresponding futures contracts. Data is sourced from Bloomberg, and we use the first nearby generic commodity futures as historical time series of returns.[6] For example, we use the ticker "NG1 Comdty" for trading natural gas. Further, to ensure that we only consider returns of liquidly traded commodity futures contracts, we align the start date of each commodity with the start date of the corresponding S&P GSCI single commodity index.

Coming back to the commodity index considered, we see how the GSCI puts a high weight on oil and gas compared to other major commodity indices, such as the Dow Jones UBS Commodity Index or the UBS Bloomberg CMCI. The annualized excess return of the GSCI, measured from January 1983 to December 2014, amounts to 2.8% at a volatility of 19.6%, which implies a Sharpe ratio of 0.14. Among the constituent commodities there are multiple time series with more sizable volatility figures, see Table 1. The range is from 14.0% (live cattle) to 58.7% (natural gas). Likewise, the range of annualized returns is quite large. Brent and unleaded gas oil show the highest excess return (18.1%), while Natural Gas had the lowest return (-5.7%). Across the board, we note that investing in single commodities entails significant downside risk. The maximum 1 year loss within the three decades covered by our data ranges from -22.7% (feeder cattle) to -84.2% (natural gas).

[5] See Table 1 for the complete list of the underlying commodities considered.

[6] Commodity futures are combined via the Bloomberg default roll-over methodology available in the GFUT function of Bloomberg. The default settings impose the roll-over of a future contract to the next expiring contract at the beginning of the contract month, or alternatively at the last trading day, as indicated in Bloomberg, should this happen before the contract month. Further, to obtain an excess return view on the commodity the time series of returns are backward adjusted by selecting the option "adjust for ratio" in the GFUT function of Bloomberg.

Table 1

Descriptive Statistics of the GSCI Commodity Universe

The table lists the descriptive statistics for the 24 commodities of the S&P GSCI as of 2014. The corresponding Bloomberg tickers and sectors are given, together with the start date of the underlying time series (according to the start date of the corresponding S&P GSCI Index). The Bloomberg tickers are reported via short name. For example, for Natural Gas the short name NG represents the Bloomberg ticker NG1 Comdty. The target weights, as of mid-2014, are given for the S&P GSCI, the UBS Bloomberg CMCI, and the Dow Jones UBS Commodity Index. The rightmost panel gives performance and risk statistics of each commodity. Annual average return and volatility figures are reported, together with the corresponding Sharpe ratio. The value at risk (VaR) and expected shortfall (ES) are computed at the 95% confidence interval over a one-year period. Maximum drawdown (MDD) is computed over a time-period of one year as well. For each commodity, all available data is used. The maximum time window is from January 1983 to December 2014.

Commodity	Ticker	Sector	Start	Index Weights			Return	Volatility	SR	VaR	ES	MDD
				CMCI	*GSCI*	*DJ UBS*				95%	95%	
WTI	CL	Crude Oil	Jan '87	10.4%	23.7%	8.5%	12.5%	34.8%	0.36	-44.8%	-59.3%	-66.4%
Brent	CO	Crude Oil	Jan '99	9.6%	23.1%	6.5%	18.1%	33.8%	0.53	-37.5%	-51.6%	-68.0%
Heating Oil	HO	Refined Products	Jan '83	4.2%	6.0%	3.7%	16.0%	35.5%	0.45	-42.3%	-57.1%	-63.2%
Gas Oil	QS	Refined Products	Jan '99	4.9%	8.3%	3.6%	18.1%	32.4%	0.56	-35.2%	-48.7%	-62.4%
RBOB	XB	Refined Products	Jan '88	5.1%	5.9%	0.0%	11.7%	35.3%	0.33	-46.4%	-61.1%	-63.1%
Natural Gas	NG	Natural Gas	Jan '94	4.0%	2.6%	9.4%	-5.7%	58.7%	-0.10	-100%*	-100%*	-84.2%
Copper	LP	Industrial Metals	Jan '83	12.4%	3.2%	7.5%	9.5%	26.3%	0.36	-33.7%	-44.7%	-58.4%
Zinc	LX	Industrial Metals	Jan '91	2.2%	0.5%	2.3%	0.9%	27.3%	0.03	-44.1%	-55.5%	-62.3%
Aluminium	LA	Industrial Metals	Jan '91	6.1%	2.0%	4.7%	-2.7%	19.8%	-0.14	-35.3%	-43.6%	-60.6%
Nickel	LN	Industrial Metals	Jan '93	2.2%	0.5%	2.1%	11.6%	36.1%	0.32	-47.7%	-62.8%	-69.4%
Lead	LL	Industrial Metals	Jan '95	1.4%	0.4%	0.0%	9.3%	30.1%	0.31	-40.2%	-52.7%	-69.1%
Gold	GC	Precious Metals	Jan '83	4.4%	2.8%	11.5%	-0.3%	15.9%	-0.02	-26.5%	-33.1%	-34.1%
Silver	SI	Precious Metals	Jan '83	1.1%	0.4%	4.1%	-0.9%	28.6%	-0.03	-47.9%	-59.9%	-51.1%
Wheat	W	Grains	Jan '99	2.2%	3.5%	3.3%	-3.7%	26.4%	-0.14	-47.1%	-58.1%	-57.3%
Kansas Wheat	KW	Grains	Jan '99	1.0%	0.8%	1.2%	1.4%	29.3%	0.05	-46.8%	-59.1%	-54.6%
Corn	C	Grains	Jan '83	6.1%	4.9%	7.2%	-2.6%	26.2%	-0.10	-45.8%	-56.7%	-58.0%
Soybeans	S	Grains	Jan '83	9.6%	2.9%	11.2%	5.1%	24.2%	0.21	-34.8%	-44.9%	-43.9%
Sugar	SB	Softs	Jan '83	6.7%	1.5%	4.0%	4.3%	38.7%	0.11	-59.4%	-75.6%	-75.2%
Cocoa	QC	Softs	Jan '84	0.0%	0.2%	0.0%	-2.8%	27.9%	-0.10	-48.7%	-60.4%	-51.3%
Coffee	KC	Softs	Jan '83	1.0%	0.6%	2.3%	2.0%	37.5%	0.05	-59.6%	-75.3%	-61.3%
Cotton	CT	Softs	Jan '83	1.3%	1.0%	1.6%	2.7%	27.0%	0.10	-41.7%	-53.0%	-57.8%
Feeder Cattle	FC	Livestocks	Jan '02	0.0%	0.5%	0.0%	6.2%	15.1%	0.41	-18.7%	-25.1%	-22.7%
Live Cattle	LC	Livestocks	Jan '83	2.3%	2.8%	3.3%	4.7%	14.0%	0.33	-18.4%	-24.3%	-28.1%
Lean Hog	LH	Livestocks	Jan '83	1.8%	1.7%	1.9%	2.9%	25.8%	0.11	-39.5%	-50.3%	-54.2%

*Figure set to -100% if the parametric value at risk or expected shortfall are below -100%.

To get a first idea of the diversification potential inherent in the commodities universe, we investigate the average correlation structure during the sample period from 1983 to 2014. In particular, Table 2 reduces the full correlation matrix to a sector correlation matrix, giving the average within-sector and between-sector correlations of the eight main commodity sectors corresponding to the 24 commodities. The within-sector correlations are calculated by averaging the pairwise correlations among all commodity futures, in each sector for each year in our sample period.[7]

Table 2
Commodity Sector Correlations

The table summarizes the average within- and between-sector correlations of the 24 commodities of the S&P GSCI grouped in eight commodity sectors. The within-sector correlations are calculated by averaging the pairwise correlations across all commodity futures, in each sector, for each year, of the sample period. The between-sector correlations are obtained by averaging the correlations between individual futures in the two sectors for each year of the sample period from January 1983 to December 2014.

Commodity Sector	Crude Oil	Ref. Prod.	Natural Gas	Industrial Metals	Precious Metals	Grains	Softs	Live Stocks
Crude Oil	0.96							
Refined Products	0.83	0.90						
Natural Gas	0.29	0.35	1.00					
Industrial Metals	0.29	0.29	0.05	0.65				
Precious Metals	0.22	0.24	0.05	0.29	0.86			
Grains	0.11	0.16	0.12	0.19	0.19	0.72		
Softs	0.07	0.10	-0.01	0.17	0.12	0.18	0.32	
Livestocks	0.06	0.08	-0.05	0.04	-0.05	-0.04	-0.02	0.65

The between-sector correlations for any pair of groups is obtained by averaging the correlations between individual futures in both groups over each year in our sample period. While all of the within-sector correlations are generally high, the between-sector correlations most often are not, which confirms the findings of Vrugt, Bauer, Molenaar, and Steenkamp (2007). On the one hand, the most heterogeneous commodity sector is softs, given the within-correlations of 0.32. On the other hand, the energy and metals sectors prove to be more in sync, given the within-correlations close to one. Between sectors, livestock, as well as softs, are hardly correlated to anything else.

[7]While the correlation matrix is based on custom commodity returns, our findings are consistent with the correlation matrix arising from sector indices as available from S&P GSCI.

Most of the remaining between-sector correlations are lower than 0.3, with the exception of the one between crude oil and refined oil (0.83). These preliminary results suggest that there is ample room for diversification within the universe of commodities.

3.2 Principal portfolios and minimum torsions

In theory, one can construct as many principal portfolios as assets entering the PCA decomposition. However, it is already well known that a small number of principal portfolios is sufficient to explain most of the assets' variation. We compute the 24 principal portfolios pertaining to the GSCI by performing a PCA over an expanding window, starting with 36 months of observations.

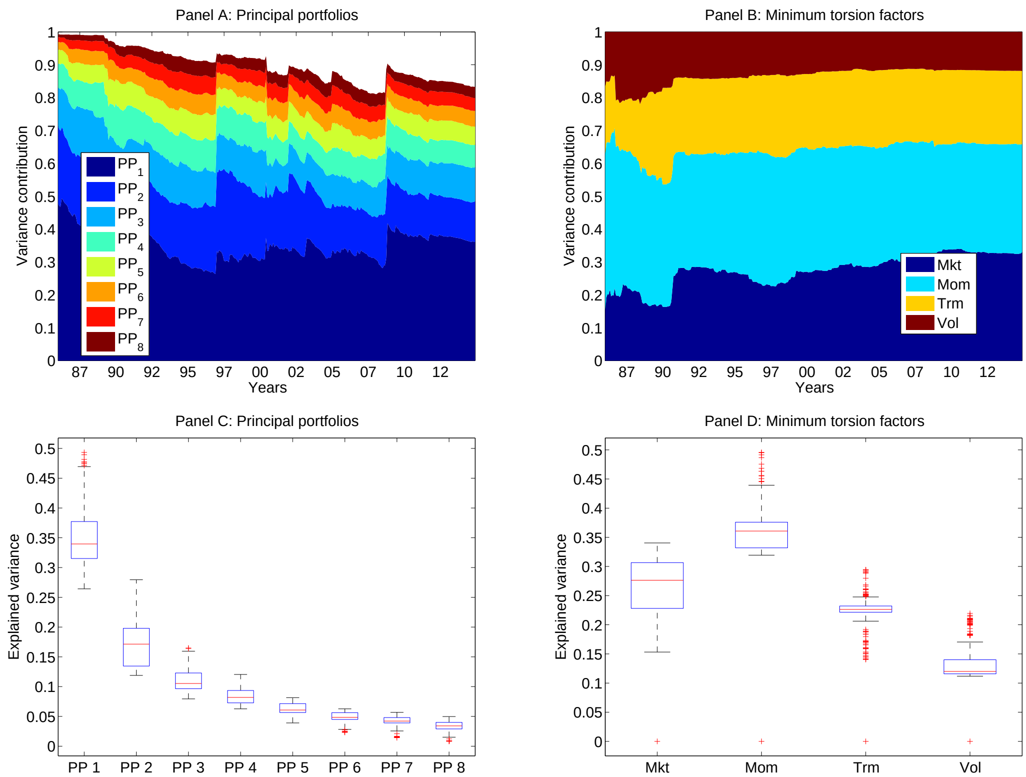
In Panels A and C of Figure 1, we assess the relevance of the principal portfolios over time. We observe that Principal Portfolio 1 (PP1) typically accounts for around 34% of total asset variability.

PP2 captures 18% on average, while PP3 captures 11%, thus leaving only single-digit fractions for the subsequent principal portfolios PP4 to PP24. Of course, with their relevance quickly dying off, it seems hardly reasonable to allocate any risk budget to higher principal portfolios. For the empirical analysis, we decided to fix the first eight principal portfolios. These account for at least 80% of the dataset's variation and is reflective of the 24 commodity assets falling into eight, rather low correlated, industrial sectors.

While the first principal portfolios are designed to capture most of the commodities' variance, these factors are of purely statistical nature. Alternatively, one could resort to factors with a sound economic rationale. Of course, such factors usually do not display zero correlation. Yet, applying the minimum torsion of Meucci, Santangelo, and Deguest (2014) allows to de-correlate these factors while sticking to the original factors as close as possible. From a risk factor view, the commodity universe gives rise to a few persistent commodity risk factors.

Figure 1. Principal Portfolios vs. Minimum Torsions

The upper charts give the variance of the principal portfolios and the minimum torsions, as well as their relative decomposition over time. The lower charts give the box-plots pertaining to a given principal portfolios or minimum risk factor torsions explained fraction of total variance over time. The results range from January 1986 to December 2014. The risk factors are market (Mkt), momentum (Mom), term structure (Trm), and volatility (Vol) factor.



The work of Miffre and Rallis (2007), Basu and Miffre (2013), and Fuertes, Miffre and Rallis (2010, 2013) identifies four main commodity risk factors: a market risk factor, a momentum factor, a term structure factor, and a volatility factor. In this vein, we employ a commodity factor model, where the excess return of the GSCI relative to the risk-free rate serves as the market return. In addition, we construct three long-only factors as follows. At any given re-balancing date, we sort the GSCI constituents according to the factor's defining criterion, and equally invest in the commodities corresponding to the top one third.[8] For instance, the momentum factor is long the

[8]Note that factor portfolios therefore build on 4 commodities at the beginning of the sample period. This number gradually increases over time up to a maximum of 8 commodities at the end of the sample period.

top one third of the best performing commodities, as measured by the commodities' past three months' return. As for the term structure factor, we rank commodities according to the steepness of their corresponding term structure, where steepness is simply the relative difference between the third and first nearby future contracts. The term structure factor is long the top one third of commodities in normal backwardation. Hence, the term structure factor identifies strategies which overweight commodities with more favorable term structures. Finally, the volatility factor invests in commodities that experience low volatility over a prolonged period of time. This factor sorts commodities in ascending order and invests in the top one third, thereby representing the commodities with the lowest historical volatility as measured over three years of monthly return data.

Table 3
Commodity Factor Correlations

The table summarizes the return, volatility, Sharpe ratio, and correlations of the commodity factors market, momentum, term structure, and volatility, as well as the loadings in the corresponding minimum risk factor torsions during January 1986 to December 2014.

Commodity Factors	Market	Momentum	Term Structure	Volatility
Panel A: Risk and Return Figures				
Return p.a.	3.1%	12.4%	9.6%	2.9%
Volatility p.a.	20.3%	19.3%	17.9%	11.7%
Sharpe Ratio	0.15	0.64	0.54	0.25
Panel B: Correlations				
Market	1.00	0.68	0.72	0.41
Momentum	0.68	1.00	0.75	0.52
Term Structure	0.72	0.75	1.00	0.50
Volatility	0.41	0.52	0.50	1.00
Panel C: Minimum Torsion Loadings				
MT Market	1.42	-0.30	-0.49	-0.10
MT Momentum	-0.29	1.50	-0.50	-0.34
MT Term Structure	-0.38	-0.42	1.59	-0.28
MT Volatility	-0.04	-0.13	-0.13	1.17

Table 3 summarizes the risk-return profile of the chosen commodity factor model, as well as their loadings in the minimum torsion factors, for the sample period from January 1986 to December 2014. The factors show a positive average yearly return, which ranges from 2.9% for the volatility portfolio to 12.4% of the momentum factor. By construction, the volatility factor results in the lowest volatility of 11.7% across the four factors. All commodity factors' Sharpe ratios are positive. The momentum portfolio has the highest Sharpe ratio (0.64), followed by the term structure (0.54), and the volatility portfolios (0.27). The market portfolio's (as represented by the S&P GSCI Excess Return index) has the lowest Sharpe ratio (0.15) mainly due to the high volatility of its oil constituents. The correlation between the commodity factors is considerably positive, and ranges from 0.41 (market vs. volatility) to 0.75 (momentum vs. term structure). In Table 3 we also highlight the loadings of the minimum risk factor torsions with regard to the original commodity factor model. Every minimum risk factor torsion strongly loads on the respective factor to be mimicked and leverages the remaining factors to ensure orthogonality.

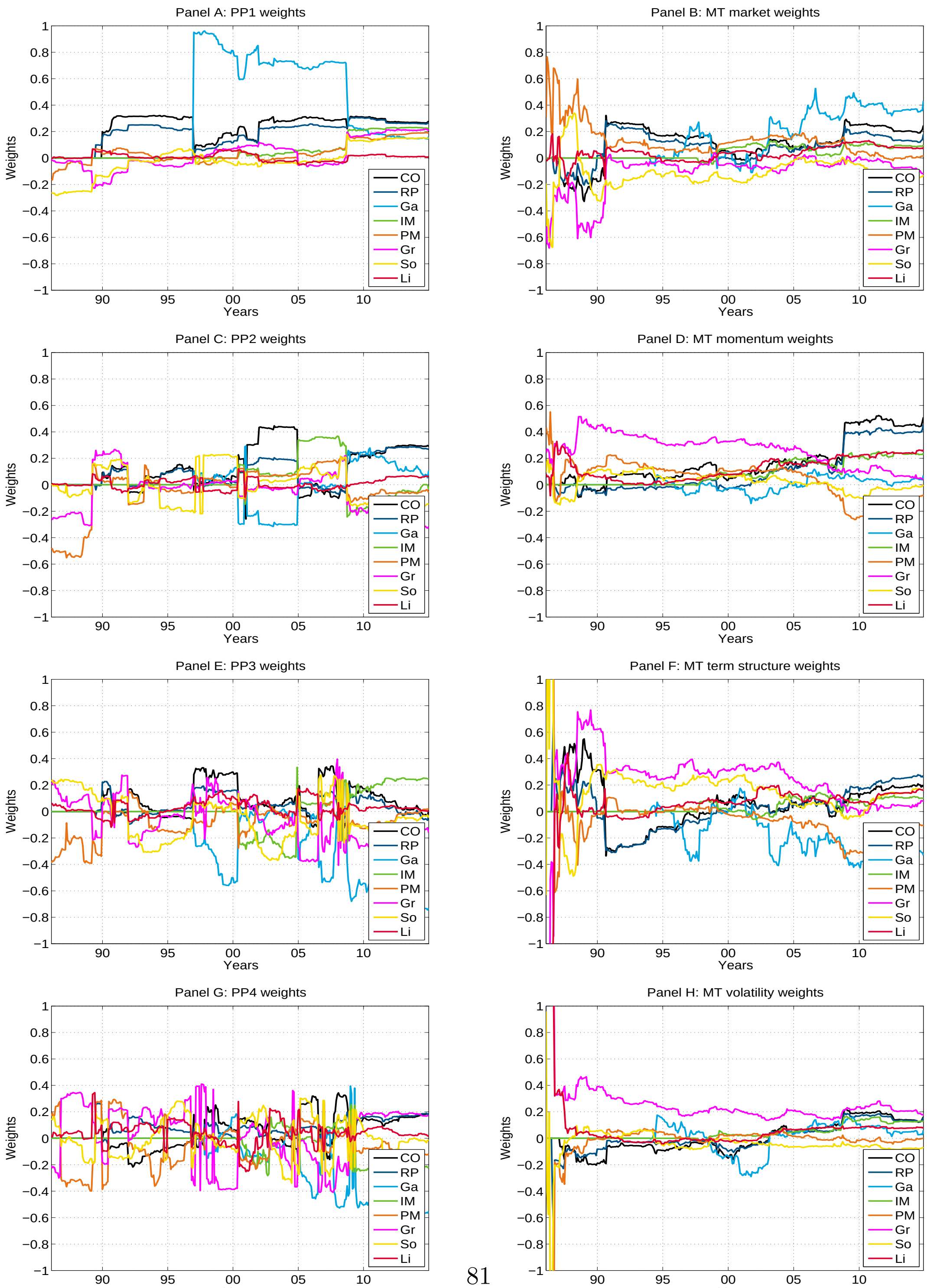
In Panels B and D of Figure 1 we assess the risk decomposition of assets' variability with respect to a factor model driven by the minimum risk factor torsions. The orthogonalized market factor (Mkt) accounts for 28% of total variability, the momentum factor (Mom) accounts for 36%, followed by the term structure factor (Trm) with 24% of total explained variance on average, and the volatility factor (Vol) explains 14%. Comparing the two alternative risk decompositions in Panels A and B of Figure 1, we observe that the percentage of explained variance is considerably more stable under the minimum torsion approach than the PCA one which, from a practical view-point, is a highly desirable property.

3.3 Rationalizing principal portfolios and minimum risk factor torsions

To foster intuition with respect to the principal portfolios relative to the minimum risk factor torsions, we investigate the single commodity loadings (or weights) of these two sets of uncorrelated risk sources. Figure 2 plots these weights for the first four principal portfolios (in the left column) and for the four minimum risk factor torsions (in the right column).

Figure 2. Principal Portfolio vs. Minimum Torsion Weights

The figures on the left give the principal portfolio weights over time. The figures on the right show the minimum torsion weights over time. The results are obtained using an expanding estimation window. The results range from January 1986 to December 2014. Commodity sectors are crude oil (CO), refined products (RP), gas (Ga), industrial metals (IM), precious metals (PM), grains (Gr), softs (So), and livestock (Li).



Both the principal portfolio 1 (PP1) in Panel A of Figure 2 and the first minimum risk factor torsion (MT_{Market}) in Panel B qualify for a common commodity risk factor with a net positive weight for most constituent commodity assets. While this effect holds for PP1 by imposing it to load on positive risk premiums only, this effect arises naturally for MT_{Market} that tracks the market risk factor provided by the S&P GSCI. Unsurprisingly, both factors load more heavily towards highly volatile commodity sectors like energy and metals rather than livestock. Beyond PP1, the principal portfolios are less straightforward to interpret. Conversely, $MT_{Momentum}$ tracks the commodity momentum factor and loads positively on oil and refined products, and it loads negatively on precious metals and softs. MT_{Term} tracks the term structure factor. It is long refined products, metals and short crude oil, natural gas, and grains. $MT_{Volatility}$ tracks the volatility factor and trades grains against softs and livestock. Figure 2 further confirms the common objection with respect to statistical risk factors derived from the principal component analysis, namely the instability of factors over time. Clearly, minimum risk factor torsions provide a more stable uncorrelated decomposition of the commodity asset universe over time. This is even more so the case after the 90s, when more data is available. In contrast, principal portfolio weights still change sign even towards the end of the sample period.

4 Diversified Commodity Investing

We next examine the empirical performance and risk profile of diversified risk parity strategies along principal portfolios or minimum risk factor torsions vis-à-vis selected benchmarks in a long-only setup. Given that the first PCA estimation as well as the computation of the minimum risk factor torsions consumes 36 months of data, the strategy performance can be assessed from January 1986 to December 2014.

Table 4
Performance and Risk Statistics of Risk-Based Commodity Strategies

The table gives performance and risk statistics of the risk-based commodity strategies from January 1986 to December 2014. Four versions of DRP strategies are considered: two DRP strategies, which diversify by principal portfolios (PP); and two DRP strategies, which diversify by minimum risk factor torsions (MT). For each type of DRP strategy, we report risk figures of the optimal unconstraint strategy (Opt.) and those of the same strategy under long-only constraints (Con.). Annual return and volatility figures are reported, together with the Sharpe ratio. Annual value at risk and expected shortfall are computed at the 95% confidence level. One year maximum drawdown (Max. DD) is reported. Turnover is the portfolios mean monthly turnover over the whole sample period. Gini coefficients are reported for portfolios weights ($Gini_{Weights}$), risk decomposition with respect to the underlying asset classes ($Gini_{Risk}$), principal portfolios ($Gini_{PPRisk}$), and minimum factor torsions ($Gini_{MTRisk}$). The $\#PP_{bets}$ is the exponential of the risk decomposition's entropy, when measured against the principal portfolios, and $\#MT_{bets}$, when measured against the minimum risk factor torsions.

Statistic	**Diversified Risk Parity**				**S&P GSCI**	**Risk-Based Allocations**			
	PP		*MT*			*1/N*	*MV*	*RP*	*MDP*
	Opt.	*Con.*	*Opt.*	*Con.*					
Panel A: Risk and Return Figures									
Return	-1.5%	7.3%	5.5%	5.1%	3.1%	3.9%	3.8%	3.5%	2.8%
Volatility	10.4%	16.4%	12.1%	12.3%	20.3%	12.9%	9.7%	10.8%	11.2%
SharpeRatio	-0.15	0.44	0.46	0.42	0.15	0.31	0.40	0.32	0.25
VaR 95%	-18.7%	-19.7%	-14.4%	-15.1%	-30.3%	-17.3%	-12.1%	-14.3%	-15.6%
ES 95%	-23.0%	-26.5%	-19.4%	-20.3%	-38.7%	-22.7%	-16.2%	-18.8%	-20.3%
Max. DD	-31.6%	-37.2%	-40.9%	-43.4%	-60.4%	-45.5%	-25.2%	-39.4%	-40.0%
Panel B: Weights and Risk Decomposition Characteristics									
Turnover	22.2%	23.5%	2.6%	3.6%	0.4%	0.3%	3.9%	1.6%	4.3%
$Gini_{Weights}$	0.34	0.65	0.27	0.41	0.53	0.00	0.58	0.25	0.44
$Gini_{Risk}$	0.51	0.65	0.40	0.51	0.70	0.32	0.58	0.00	0.39
$Gini_{PPRisk}$	0.00	0.17	0.37	0.26	0.51	0.41	0.21	0.29	0.24
$Gini_{MTRisk}$	0.42	0.31	0.00	0.01	0.52	0.24	0.22	0.18	0.20
$\#PP_{bets}$	8.00	6.25	3.95	4.77	2.46	3.28	5.99	4.62	5.59
$\#MT_{bets}$	2.53	3.06	4.00	3.99	2.17	3.43	3.55	3.63	3.56

Panel A of Table 4 gives performance and risk statistics of the risk-based commodity strategies. Across the board we find that the classic strategies yield similar annual returns. Unsurprisingly, the lowest annual volatility (9.7%) is achieved by the minimum-variance strategy, together with an annual return of 3.8%, which compares to 3.1% for the S&P GSCI. Also, its maximum drawdown (-25.2%) is relatively small when compared to the one of the index (-60.4%).

The volatility of $1/N$ is higher (12.9%) than that of the minimum-variance strategy, but it is still smaller than the energy-induced GSCI volatility (20.3%). Also, the return of the $1/N$ strategy amounts to 3.9%, resulting in a Sharpe ratio of 0.31. Hence, the risk-return profile of the GSCI is inferior to the one given by the simple $1/N$ strategy. Reiterating Maillard, Roncalli, and Teiletche (2010), we find that the traditional risk parity strategy is a middle-ground portfolio between $1/N$ and minimum-variance. Its return is 3.5% at a volatility of 10.8%, giving rise to a Sharpe ratio of 0.32, which falls short of 0.40 for minimum-variance. Also, its maximum drawdown statistics are slightly reduced when compared with the $1/N$-strategy. The MDP fares similarly to the risk parity strategy, giving slightly less return (2.8%) at higher volatility (11.2%).

Having recovered the well-known risk and return characteristics of the classic risk-based strategies, we inspect the diversified risk parity strategies. In particular, we look at two distinct versions of DRP strategies which derive from either diversifiying by principal portfolios or by minimum risk factor torsions. For both versions, we investigate the optimal strategy (that might have short positions) as well as a constrained long-only version.

Diversifying across principal portfolios, the optimal DRP_{PP} strategy earns -1.5% at 10.4% volatility. Restricting DRP_{PP} to positive weights only curbing the strategy performance to give 7.3% return at 16.4% volatility. These figures correspond to a Sharpe ratio of 0.44. Note that the constrained DRP_{PP} entails the largest turnover among the risk-based commodity strategies with 23.5%, suggesting that transaction costs may reduce the relative return potential. Conversely, diversifying across minimum risk factor torsions is not associated with an excessive turnover for the DRP_{MT}—neither in the optimal (2.6%) nor the constrained version (3.6%). Moreover, the strategy earns an average return of 5.5% at 12.1% volatility, thereby giving a Sharpe ratio of 0.46 in the unconstrained optimal version. Enforcing long-only constraints, the risk-return characteristics of the DRP_{MT} hardly change. Its return of 5.1% at 12.3% volatility results in a Sharpe ratio of 0.42.

4.1 Risk and diversification characteristics

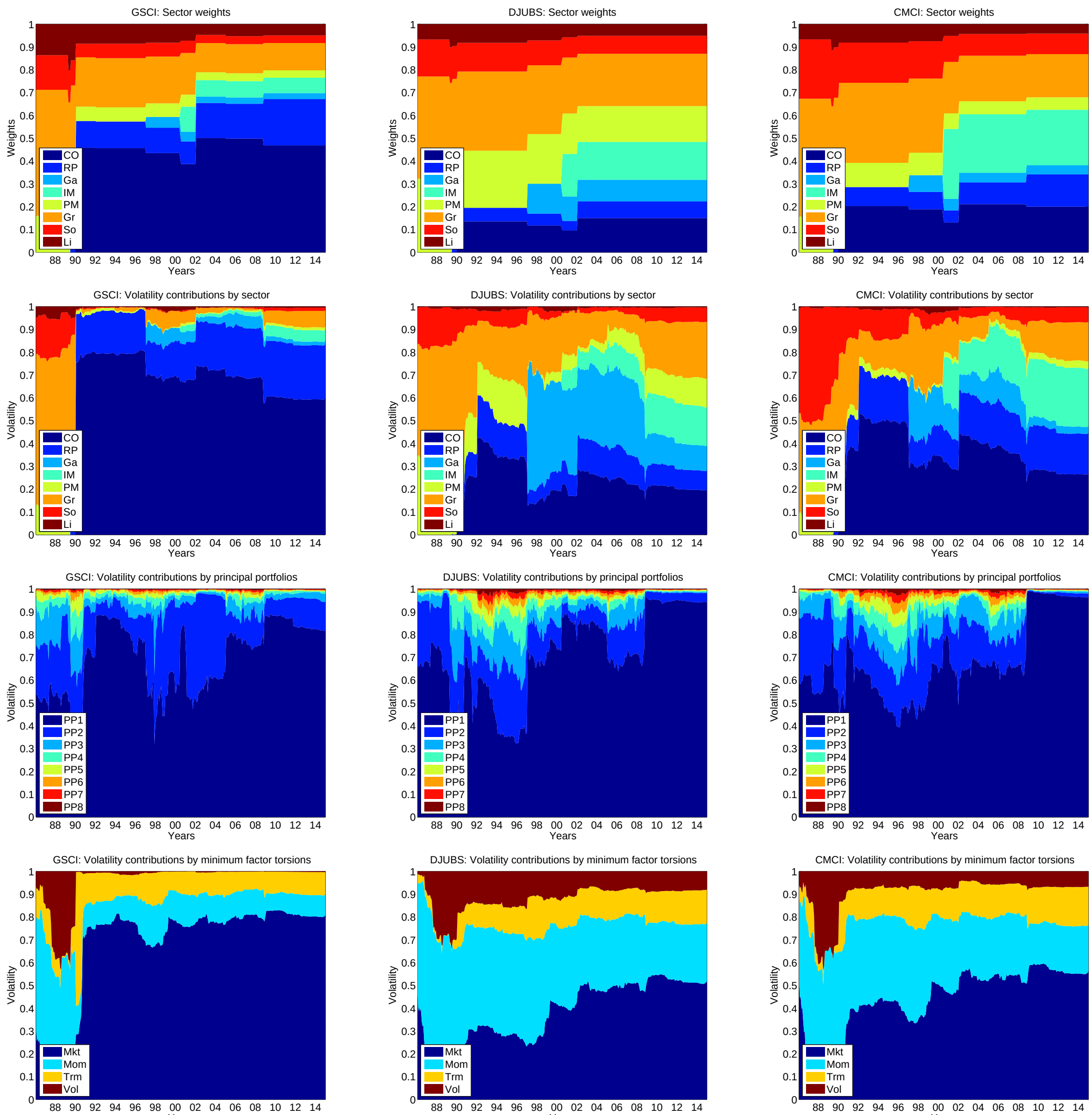
Judging risk-based strategies by their Sharpe ratios alone is not meaningful given that returns are not entering the respective objective functions. In a similar vein, we resort to evaluating the strategies along their risk and diversification characteristics. We first decompose risk by the underlying commodities, by the principal portfolios, and by the minimum risk factor torsions. This approach provides us with a concise picture of the underlying risk structure and the number of uncorrelated bets according to Equation (4) implemented in a given portfolio. The results are reported in Panel B of Table 4.

To set the stage, we start by analyzing the GSCI and provide some aggregate figures summarizing the characteristics of the weight decomposition on the commodity level. In Panel B of Table 4, we report different Gini coefficients. The Gini coefficient is a measure of concentration, which is zero in case of no concentration (equal weights throughout time) and 1 in case of full concentration (one commodity, principal portfolio, or minimum risk factor torsion attracts all of the weight all the time). Therefore, the Gini coefficient serves as a diversification measure in its own right. We can calculate Gini coefficients related on the risk decomposition by commodities ($Gini_{Risk}$), the risk decomposition by PPs ($Gini_{PPRisk}$), and the one by minimum risk factor torsions ($Gini_{MTRisk}$), respectively. For the GSCI, the $Gini_{Weights}$ (0.53) and the $Gini_{Risk}$ (0.70) show the index to be rather concentrated.

Unreported results confirm that the GSCI weights decomposition was dominated by softs and grains in the mid-80s, and it slowly evolved into an energy-driven index. Moreover, according to the risk decomposition by sectors, crude oil absorbs more than half of the risk budget most of the time. In a similar vein, the GSCI is almost exclusively exposed to the single risk factor PP1 which, typically accounts for 50% to 90% of the GSCIs total risk over time. Unsurprisingly, this result is reinforced when risk is decomposed with respect to the minimum risk factor torsions. Interestingly, this verdict seems to apply for the two other major commodities as well, the UBS Bloomberg Constant Maturity Commodity Index and the Dow Jones UBS Commodity Index as depicted in Figure 3. Even though these indices display a more diverse weights allocation, this observation does not translate into a diverse risk allocation. Conversely, we document these indices to emerge as one-bet strategies in most recent times.

Figure 3. Weights and Risk Decompositions: Commodity Indices

The figure gives the decomposition of the S&P GSCI, Dow Jones UBS, and CMCI commodity indices in terms of weights and risk. Indices are approximated by considering the commodities and weights reported in Table 1. Risk is decomposed by asset classes, by principal portfolios, and by minimum risk factor torsions, respectively. The first column contains the results for the S&P GSCI, the second column for Dow Jones UBS the third column for the CMCI. The sample period is from January 1986 to December 2014. Commodity sectors are crude oil (CO), refined products (RP), gas (Ga), industrial metals (IM), precious metals (PM), grains (Gr), softs (So), and livestock (Li).



In Table 4 we also report the Gini coefficients of the other strategies. By definition, the $1/N$ strategy has a $Gini_{Weights}$ of zero, but it has a substantial $Gini_{PPRisk}$ of 0.41, almost as high as the $Gini_{PPRisk}$ of the GSCI (0.51). Similarly, the $Gini_{PPRisk}$ and $Gini_{MTRisk}$ of the optimal DRP strategies are zero for the DRP_{PP} and DRP_{MT}, respectively. They increase slightly for the constrained version of the respective strategies.

Figures 4 and 5 depict sector weights and risk decompositions for $1/N$, the minimum-variance strategy, traditional risk parity, MDP, and the diversified risk parity strategies. For the $1/N$ strategy, the risk decomposition by principal portfolios almost collapses into a single-coloured square, which indicates that this portfolio is mainly exposed to market risk as represented by PP1. Ideally, a portfolio that reflects eight uncorrelated bets should exhibit a risk parity profile along the PPs, i.e. the decomposition should follow a constant 1/8 risk budget allocation over time.

The weights decomposition of minimum-variance is concentrated into a few assets because the strategy is collecting the lowest volatility assets. In terms of commodity sector composition, the minimum-variance strategy is overweighting more defensive sectors like softs and livestock; its risk decomposition by principal portfolios is more diverse than the one for $1/N$ or the index. Still, PP1 explains around 60% of the total risk on average. As for the traditional risk parity strategy, the weights decomposition is less concentrated, as is clear from an average $Gini_{Weights}$ of 0.25. However, its risk decomposition by PPs merely indicates 4.62 out of eight bets on average. The MDP (5.59) is similar to MV, which is slightly more diversified with 5.99 bets on average. Given that all the classical risk-based strategies load heavily on the common risk factors, we are especially interested in testing whether the DRP strategy provides a more diversified risk profile. When compared to other strategies, the DRP_{PP} strategy seems actively reallocating across sectors, see Figure 5.

Figure 4. Weights and Risk Decompositions: Risk-Based Commodity Strategies
The figure gives the decomposition of the risk-based commodity strategies in terms of weights and risk. Risk is decomposed by asset classes, by principal portfolios, and by minimum risk factor torsions, respectively. The first column contains the results for the $1/N$-strategy, the second column for minimum-variance, the third column for the MDP. The sample period is from January 1986 to December 2014. Commodity sectors are crude oil (CO), refined products (RP), gas (Ga), industrial metals (IM), precious metals (PM), grains (Gr), softs (So), and livestock (Li).

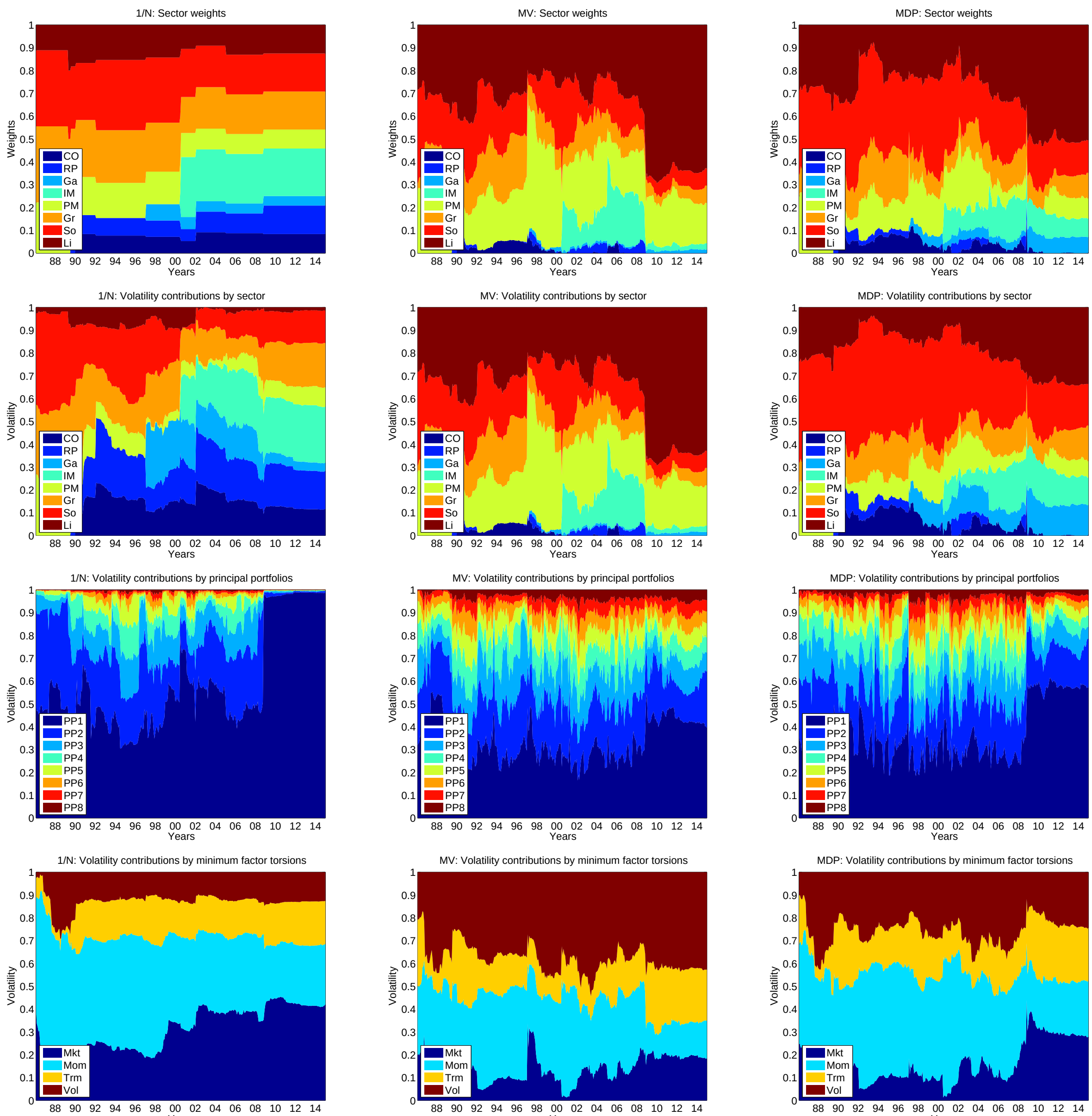
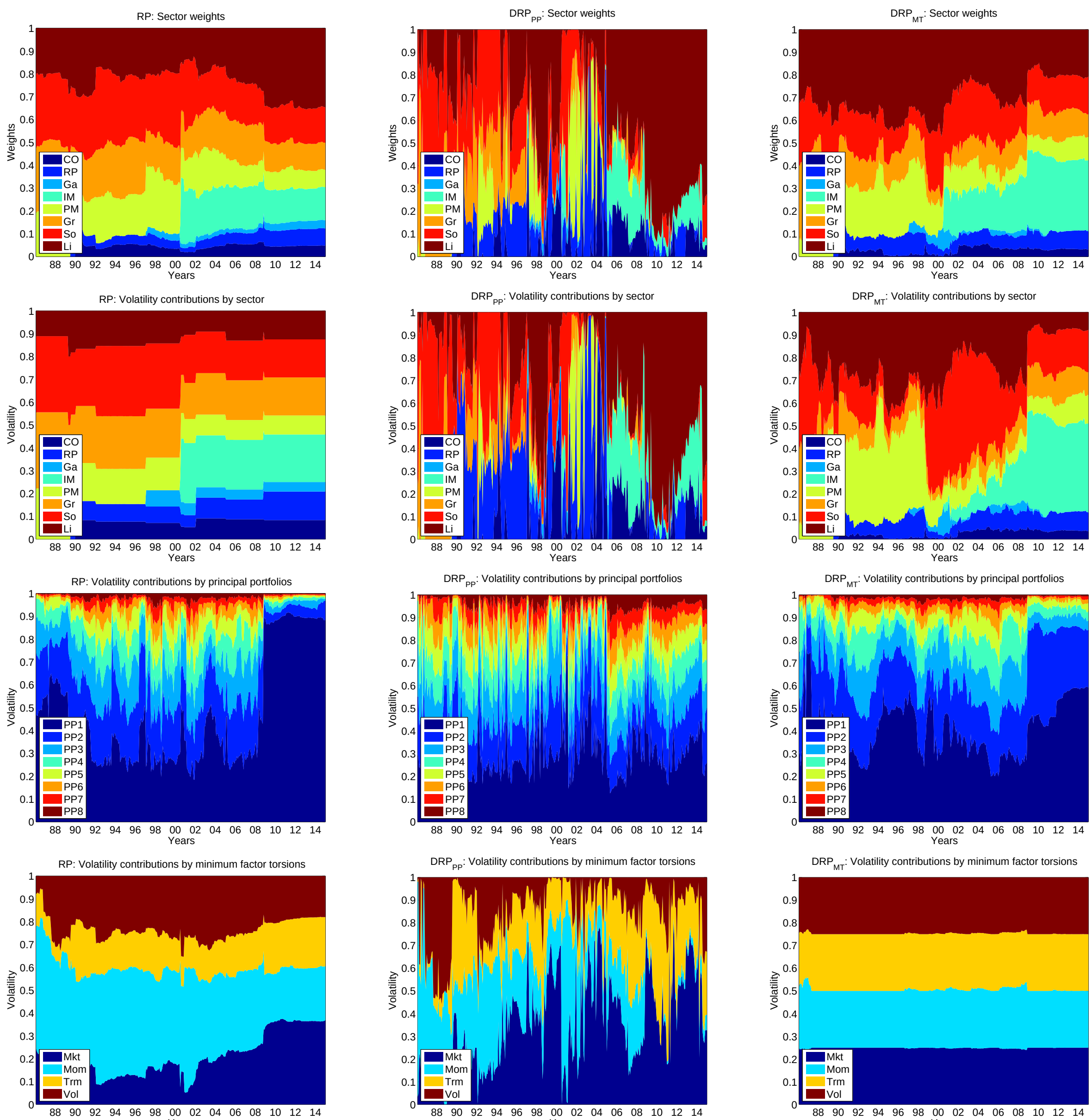


Figure 5. Weights and Risk Decompositions: Risk Parity Strategies
The figure decomposes risk parity strategies in terms of weights and risk. Risk is decomposed by asset classes, by principal portfolios, and by minimum risk factor torsions, respectively. The first column contains the results for traditional risk parity, the second column is for diversified risk parity along the principal portfolios, and the third column is for diversified risk parity along minimum torsions. The sample period is from January 1986 to December 2014. Commodity sectors are crude oil (CO), refined products (RP), gas (Ga), industrial metals (IM), precious metals (PM), grains (Gr), softs (So), and livestock (Li).

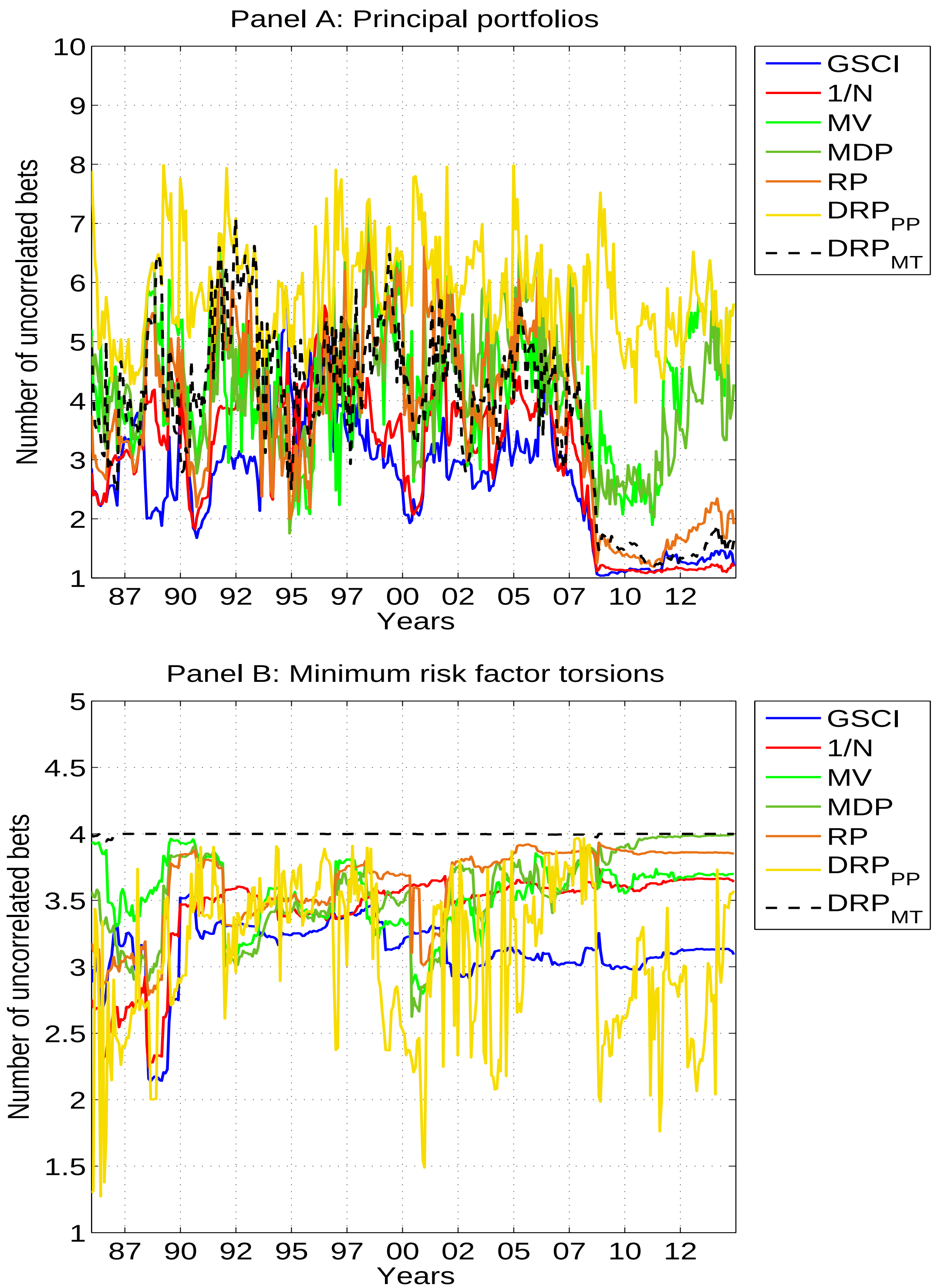


More importantly, the common risk factor represented by PP1 has lost its dominance on the risk budget, which is reflected by the 6.25 bets on average in Table 4. The DRP_{PP} strategy's combination of concentrated positioning, together with its active repositioning over time, seems to be the key for maintaining a fairly balanced risk decomposition across the uncorrelated risk sources. The DRP_{MT} can be considered as a middle-ground portfolio between rather concentrated strategies, such as the GSCI and the $1/N$-strategy, and the DRP_{PP}. Its number of bets, measured across PPs (4.77), is comparable to one of the traditional RP strategy at 4.62. Conversely, when risk is measured in terms of economically interpretable minimum torsions, the superior diversification of DRP_{MT} is evident given 3.99 bets out of the maximum number of four bets. Because of the long-only nature of the chosen factor model, most strategies show high values, ranging from the 3.06 bets for DRP_{PP} to 3.63 for RP. The odd one out is the GSCI with only 2.17 bets.

Figure 6 contrasts the strategies' degree of diversification over time in terms of the number of uncorrelated bets defined in Equation (4). First, note that the S&P GSCI index strategy is generally dominated by the other strategies in the sense that it has the smallest number of uncorrelated bets. While minimum-variance, MDP, and traditional risk parity represent a higher number of bets, one can observe a significant deterioration in diversification over the last decade during the sample period. As a result, $1/N$, the GSCI, and risk parity are rendered one-bet strategies in terms of principal portfolio bets. Conversely, diversified risk parity maintains the highest number of bets over time according to the number of relevant principal portfolios, see the Panel C of Figure 1. Of course, this observation is expected. On the right side of Figure 6, DRP_{MT} dominates the other strategies in terms of uncorrelated factor bets, with an average of 3.99 out of 4. Apart from the GSCI, the other strategies nevertheless exhibit a medium to high number of uncorrelated factor bets over time.

Figure 6. Number of Uncorrelated Bets

The plot gives the number of uncorrelated bets for the risk-based commodity strategies with respect to the principal portfolios (Panel A) and to the minimum risk factor torsions (Panel B). The data ranges from January 1986 to December 2014.



4.2 Dismantling risk-based commodity strategies

To further characterize the risk-based commodity strategies, we relate the strategies' returns to common risk factors. To this end, we rely on the following factor structure:

$$R_{RBS,t} = \alpha + \beta_1 R_{Market,t} + \beta_2 R_{Momentum,t} + \beta_3 R_{TermStructure,t} + \beta_4 R_{Volatility,t} + \varepsilon_t \quad (19)$$

where $R_{RBS,t}$ is the excess return of one of the risk-based strategies relative to the risk-free rate.

We report the results in Table 5. As expected, the factor coefficients for the DRP_{MT} are all significant, not only for the optimal but also for the constrained strategy. In contrast, for the DRP_{PP}, only the market and the volatility factor do seem to have a significant influence on the strategy's return beyond the 5% significance level. None of the strategies deliver positive alpha beyond the common risk factor controls. Similarly to DRP_{MT}, the $1/N$ strategy loads significantly on all common risk factors — however, with a larger load on the market factor. Furthermore, the factors explain most of the $1/N$ strategy's time series variation with an R^2 of 79.9%. The variation of the RP strategy is also well captured by the commodity factors with an adjusted R^2 of 78.2%. Rather than a uniform allocation across common factors, MV and RP show a strong tilt towards low volatility assets (loading of 0.58 and 0.54, respectively).

The DRP_{MT} strategy instead gives rise to a more balanced exposure towards the common factors, which reflects their heterogeneous risk profile, loading less on the market, momentum and term structure, and slightly more on low volatility assets. Interestingly, for all strategies, the loading on low volatility assets is significant.

Only one-third of the excess time series variation can be attributed to common factors for DRP_{PP} (32.3%), while some two-thirds of the variation of DRP_{MT}, MDP, and MV can be explained. We conjecture that the DRP might be playing commodity factors more actively than the other strategies, making it hard to pinpoint these exposures in a static time series regression.

Table 5
Time Series Regressions of Risk-Based Commodity Strategies

The table gives time series regression results according to factor model (19) for the risk-based commodity strategies using the period from January 1986 to December 2014. Four versions of DRP strategies are considered: two DRP strategies, which diversify by principal portfolios (PP); and two DRP strategies, which diversify by minimum risk factor torsions (MT). For each type of DRP strategy, we report regression results of the optimal unconstrained strategy (Opt.) and those of the same strategy under long-only constraints (Con.). Coefficients are in bold face when significant on a 5%-level; they are in italics when significant on a 10%-level. Below each coefficient the corresponding t-statistic is reported in parentheses.

Statistic	**Diversified Risk Parity**				**Risk-Based Allocations**			
	PP		*MT*		*1/N*	*MV*	*RP*	*MDP*
	Opt.	*Con.*	*Opt.*	*Con.*				
Regression Coefficients & t-Statistics								
Alpha (%)	-0.21	0.31	0.17	0.10	-0.06	0.14	-0.01	-0.04
	(-1.37)	(1.40)	(1.41)	(0.90)	(-0.60)	(1.37)	(-0.12)	(-0.31)
Market	**0.11**	**0.29**	**0.17**	**0.13**	**0.23**	0.03	**0.12**	**0.08**
	(2.73)	(5.26)	(5.62)	(4.69)	(9.69)	(1.32)	(5.89)	(2.84)
Momentum	*-0.08*	0.08	**0.07**	**0.07**	**0.17**	0.02	**0.10**	*0.06*
	(-1.72)	(1.32)	(1.97)	(2.16)	(6.22)	(0.56)	(4.24)	(1.86)
Term Str.	**0.11**	0.06	**0.08**	**0.12**	**0.06**	0.02	*0.05*	**0.09**
	(2.31)	(0.82)	(2.16)	(3.13)	(2.02)	(0.77)	(1.86)	(2.26)
Volatility	**0.17**	**0.23**	**0.46**	**0.51**	**0.44**	**0.58**	**0.54**	**0.50**
	(3.32)	(3.03)	(11.32)	(13.09)	(13.90)	(16.88)	(19.36)	(12.39)
Adj. R^2	15.9	32.3	62.3	66.5	79.9	59.1	78.2	58.0

5 Robustness checks

In this section, we perform a series of robustness checks.

5.1 Rolling window analysis

All of the presented results build on an expanding window estimation. While this approach usually introduces a meaningful degree of stability, one may argue that a rolling window analysis can be more adaptive with respect to changes in the risk structure.

Table 6
Risk-Based Commodity Strategies: Rolling window analysis

The table gives performance and risk statistics of the risk-based commodity strategies from January 1986 to December 2014, based on rolling window analysis with a 36 months estimation window. Four versions of DRP strategies are considered: two DRP strategies, which diversify by principal portfolios (PP); and two DRP strategies, which diversify by minimum risk factor torsions (MT). For each type of DRP strategy, we report risk figures of the optimal unconstraint strategy (Opt.) and those of the same strategy under long-only constraints (Con.). Annual return and volatility figures are reported, together with the Sharpe ratio. Annual value at risk and expected shortfall are computed at the 95% confidence level. One year maximum drawdown (Max. DD) is reported. Turnover is the portfolios mean monthly turnover over the whole sample period. Gini coefficients are reported for portfolios weights ($Gini_{Weights}$), risk decomposition with respect to the underlying asset classes ($Gini_{Risk}$), principal portfolios ($Gini_{PPRisk}$), and minimum factor torsions ($Gini_{MTRisk}$). The $\#PP_{bets}$ is the exponential of the risk decompositions entropy, when measured against the principal portfolios, and $\#MT_{bets}$, when measured against the minimum risk factor torsions. The table reports statistics obtained investing in S&P GSCI single commodity indices.

Statistic	**Diversified Risk Parity**				**S&P**	**Risk-Based Allocations**			
	PP		*MT*		**GSCI**	*1/N*	*MV*	*RP*	*MDP*
	Opt.	*Con.*	*Opt.*	*Con.*					
Panel A: Risk and Return Figures									
Return	3.4%	5.4%	4.7%	5.0%	3.1%	3.9%	3.4%	3.4%	2.9%
Volatility	11.2%	16.4%	12.8%	15.4%	20.3%	12.9%	9.7%	10.7%	11.5%
SharpeRatio	0.30	0.33	0.37	0.32	0.15	0.31	0.35	0.32	0.25
VaR 95%	-15.0%	-21.6%	-16.3%	-20.4%	-30.3%	-17.3%	-12.7%	-14.3%	-16.0%
ES 95%	-19.7%	-28.5%	-21.7%	-26.8%	-38.7%	-22.7%	-16.7%	-18.7%	-20.9%
Max. DD	-32.4%	-33.8%	-32.4%	-50.2%	-60.4%	-45.5%	-26.4%	-37.0%	-37.7%
Panel B: Weights and Risk Decomposition Characteristics									
Turnover	36.9%	38.4%	10.7%	22.7%	0.4%	0.3%	8.1%	3.3%	9.2%
$Gini_{Weights}$	0.33	0.65	0.30	0.43	0.53	0.00	0.58	0.27	0.48
$Gini_{Risk}$	0.50	0.64	0.46	0.52	0.69	0.31	0.58	0.00	0.43
$Gini_{PPRisk}$	0.00	0.18	0.36	0.35	0.49	0.43	0.20	0.31	0.24
$Gini_{MTRisk}$	0.43	0.37	0.00	0.07	0.52	0.29	0.26	0.22	0.23
$\#PP_{bets}$	8.00	6.00	3.74	3.95	2.74	3.12	5.94	4.54	5.67
$\#MT_{bets}$	2.47	2.81	4.00	3.92	2.11	3.18	3.41	3.47	3.41

To investigate this possibility, we have repeated the computation of the risk-based commodity strategies using a rolling window of 36 months. Table 6 gives the according strategy results. The baseline findings continue to hold, although the strategies' returns tend to be slightly smaller compared to the expanding window case. More importantly, the strategies' turnover unduly increases.

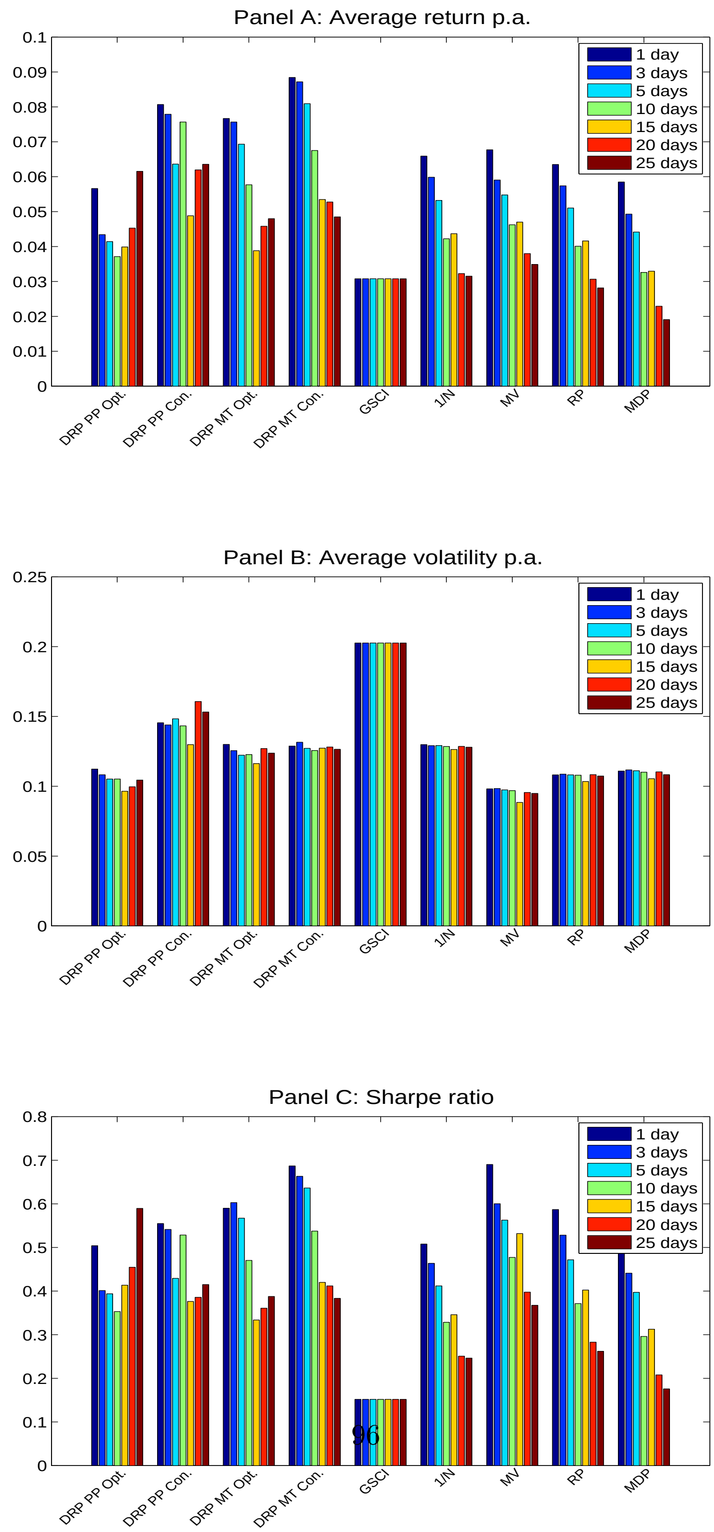
While this result obviously relates to the increased responsiveness of the strategies to changes in the risk structure, the associated increase in transaction costs will further decrease the strategies' performance. These effects are most pronounced for the DRP strategies, which should naturally benefit from a more robust estimation of the underlying commodity factor correlation structure.

5.2 Altering the underlying roll day assumption

All of the presented results depend on generic commodity future returns that are computed using the default Bloomberg settings. It is natural to investigate the strategies' results under different assumptions regarding the rolling of the contracts. In Figure 7, we plot risk and return figures and the ensuing Sharpe ratio for different roll-day assumptions ranging from 1 day to 25 days prior contract expiry. First, we note that a given strategy's return tends to increase the closer we move towards expiry. The latter finding holds for all strategies including DRP across minimum risk factor torsions but excluding DRP across principal portfolios. The latter will most likely relate to the principal portfolios' instability. Second, the volatility of the different strategies is robust to changes in the roll day. Third, the ranking of strategies in terms of risk-adjusted performance across any roll-day bucket is consistent with the baseline results of our analysis favoring diversified risk parity strategies along minimum risk factor torsions.

Figure 7. Performance of Risk-Based Commodity Strategies by Roll Day

The plot gives the average return and volatility p.a. together with the corresponding Sharpe ratio for all risk-based commodity strategies. Results are derived by rolling-over contracts X days before expiry, where X takes the values 1, 3, 5, 10, 15, 20, or 25 days. Strategy performance is evaluated over January 1986 to December 2014.



6 Conclusion

Given an increased desire for risk control emanating from the most recent financial crisis, there has been considerable interest for investors in strategies seeking for maximum diversification. As noticed in Lintner (1983), commodity futures, portfolios, indices, and CTAs provide diversification opportunities for investors due to their low correlation with respect to stocks and bonds.

Our research objective was to exploit these stylized facts when exploring new ways of commodity investing, which would provide maximum diversification along uncorrelated risk sources inherent in the commodity universe. Judging by the results from our study, diversified risk parity strategies are distinct in several aspects from the prevailing risk-based portfolio construction paradigms.

Moreover, our research has several practical implications. First, we have extracted the relevant uncorrelated risk sources embedded in a classic commodity universe and fostered intuition with respect to their economic nature. Second, the framework is a convenient risk management tool for decomposing the risk of any given strategy by uncorrelated risk sources and for assessing its degree of diversification. Third, the diversified risk parity strategies represent an innovative way of risk-based portfolio construction to generate truly diversified commodity portfolios.

Besides the above long-only strategies, the commodity futures market provides a natural environment for implementing long-short self-financing strategies. Our empirical results show that diversified risk parity strategies maintain a balanced exposure to the commodity market's uncorrelated risk sources. Allowing for negative weights, there is even more room to exploit the diversification potential via diversified risk parity strategies.

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Part IV

Second Order Risk In Alternative Risk Parity Strategies

Simone Bernardi, Markus Leippold, and Harald Lohre

Abstract

The concept of second order risk operationalizes estimation risk in portfolio construction induced by model uncertainty. In this paper we study its contribution to realized volatility of recently developed risk parity strategies. For each strategies we derive closed-form solutions of second order risk, to be illustrated in empirical analysis based on real market data. These results suggest a relation between the contribution of second order risk and the sensitivity of portfolios to single eigenvectors of the covariance matrix of assets' returns. Among the strategies considered, we find the principal risk parity strategy, that equally invests in each eigenvector underlying the variance-covariance matrix, to be immune to second order risk. For the other strategies, second order risk can be partially mitigated by means of statistical methods.

Keywords: Estimation Risk, Second Order Risk, Portfolio Construction, Risk Parity, Diversification

JEL Classification: G11; D81

1 Introduction

When modern portfolio theory emerged with the seminal paper of Markowitz (1952) on mean–variance (MV) optimization, estimation risk was mostly neglected and the estimated parameters were treated as if they were the true parameters. However, especially in finance, estimation risk is unavoidable. As indicated by a wide number of authors, such as Jobson, Korkie, and Ratti (1979), Jorion (1986), and Michaud (1989), allocating assets following the mean–variance paradigm without recognizing the existence of the estimation risk inherent in the parameters has a huge impact on the optimized portfolios and leads to several undesirable features and deficiencies, such as, unstable weights, concentrated allocations, excessive portfolio turnover, lower returns, and the realized portfolio volatility's exceeding the ex ante expected volatility.[1]

This last deficiency, a higher realized volatility than expected, has been studied in Shepard (2009), who defined a risk measure to quantify what he dubbed the Second-Order Risk (SOR) bias in optimized portfolios, i.e., a systematic deviation, induced by model uncertainty, of realized volatility from in-sample volatility. Sheppard also shows that the unconstrained minimum-variance (MV) portfolio suffers from a systematic SOR bias, which is proportional to the ratio of the number of assets to the number of in-sample observations considered in the portfolio construction. The analyzed SOR bias is consequently found to be especially pronounced for short estimation periods. Only when the number of observations is considerably greater than the number of assets does the SOR bias tend to disappear. More recently, Stefanovits, Schubiger, and Wütrich (2015) extend the empirical study of the SOR bias to the most-diversified portfolio approach of Choueifaty and Coignard (2008) and to the traditional risk parity of Maillard, Roncalli, and Teiletche (2010), with similar findings, suggesting that the SOR bias is rather a common denominator across different portfolio construction methods than an undesirable side effect limited to the MV optimization framework.

In the present paper we advance the study of SOR bias by looking at its contribution to the realized volatility of recently developed alternative risk parity strategies that allocate along

[1]See, for example, DeMiguel, Garlappi, and Uppal (2009), who empirically corroborate that some 500 years of monthly in-sample observations are necessary for the MV portfolio to outperform the $1/N$ portfolio in an asset universe of 50 U.S. stocks.

Principal Portfolios (PPs), i.e., the eigenvectors out of a principal component analysis (PCA) of the covariance matrix of the assets returns. Inspired by the work of Partovi and Caputo (2004), various authors have recognized the appealing properties of investing in terms of principal portfolios which have zero correlations by design, unlike the correlations of the underlying assets. Hence, PPs allow for a more natural description of the diversification properties of the asset universe, see Meucci (2009).

Recently, the Diversified Risk Parity (DRP) strategy, the analogue of the risk parity strategy of Maillard, Roncalli, and Teiletche (2010) but in the principal portfolio space, aims at equally weighting the contribution of each principal portfolio to the risk, where the risk is measured in terms of the volatilities of the PPs. The DRP strategy has been studied in Lohre, Neugebauer, and Zimmer (2012) with a focus on U.S. equities, and in Kind (2013) as well as Lohre, Opfer, and Ország (2014) in a multi-asset investment universe. In its optimal unconstrained version, the DRP strategy invests in each PP proportionally to the inverse of the square root of the corresponding eigenvalues, i.e., proportional to the inverse of the volatilities of the PPs. In addition, we consider two alternative variations of the DRP strategy. First, we look at the $1/V$ portfolio, which weights the PPs by the inverse of the corresponding eigenvalues, i.e., proportional to the inverse of the variances of the PPs. Second, we consider a strategy mentioned in Hall (2012). The author proposes a strategy which could be interpreted as the analogue of the equally weighted portfolio, or $1/N$ strategy, in PP space. This strategy suggests investing in the main uncorrelated risk sources in a more natural fashion than the DRP and the $1/V$ portfolio. Instead of having equal budget risks across PPs, the risk is budgeted proportionally to the contribution of each PP to the total variance. As a result, the lion's share of capital is allocated to the most significant uncorrelated risk source, that potentially carry risk premia with a higher probability, hence basically neglecting most of the less significant PPs. Technically, one is simply allocating equal weights to the PPs, which is why we call it Principal Risk Parity (PRP).

The common denominator of the considered alternative risk parity strategies is that they invest in an uncorrelated decomposition of the asset universe provided by the PCA. We note that the PCA is by no means unique in decomposing the asset universe into uncorrelated risk sources.

Leveraging on the work of Meucci, Santangelo, and Deguest (2015), various authors have proposed more sophisticated versions of alternative risk parity strategies, which we do not consider in this study. Among others, Kind and Poonia (2015) apply the minimum torsion directly to the underlying assets. Bernardi, Leippold, and Lohre (2018) start from an economically well founded commodity factor model and obtain uncorrelated risk sources by applying minimum rotations to it.

For each selected alternative risk parity strategy we derive analytical closed-form solutions for the corresponding SOR bias, assuming the assets' returns are normally distributed. The SOR bias of the $1/V$ strategy (which invests in the PPs in inverse proportion to their variance) is comparable to the SOR bias of the MV portfolio. As for the optimal DRP strategy (which instead invests in the PPs in proportion to the square root of the inverse of the corresponding principal portfolio's variances) we show that its SOR bias is approximately equal to the square root of the SOR bias of the $1/V$ and MV portfolios. This correlation between the magnitude of the corresponding SOR bias and the weight assigned to the PPs sheds light on the lower exposure of the DRP strategy to low-volatility PPs. This observation enhances expectations about an even lower SOR bias for the PRP strategy (as this portfolio invests in the PPs by equally weighting them, i.e., by allowing every single PP to contribute to the overall portfolio volatility proportionally to its own volatility). Also, the PRP strategy appears to be immune from the SOR bias and therefore not subject to systematic risk underestimation. All in all, these results shed an interesting light on the relation between the weighting of the principal portfolios and the contribution of SOR to a realized portfolio's volatility. These findings resonate well with the observation in DeMiguel, Garlappi, and Uppal (2009), who report less sensitivity to estimation risk for the correlation matrix than the covariance matrix when these are used as inputs to portfolio optimization. By equally weighting the PPs, the PRP strategy exclusively leverages the estimation of the correlation structure of the asset returns as given by the corresponding loading matrix defining the PPs. Instead, the $1/V$, DRP, and MV strategies require the additional estimation of the eigenvalues for their portfolio optimization.

Our analytical results are further confirmed by empirical analysis. We consider alternative risk parity strategies (together with the MV, the equally weighted, and a random portfolio as controls) and calculate their SOR bias over the last 10 years by running these strategies on the Fama–French industry portfolios of real equity market data as provided by the Center for Research in Security Prices (CRSP). We assess the SOR contribution to realized volatility by varying the number of assets as well as the number of in-sample observations considered. Additionally, we compare the SOR bias before and after applying estimation risk mitigation procedures to the covariance matrix. In particular, we look at simple bootstrapping, the linear shrinkage of Ledoit and Wolf (2004a), and the eigenvalue adjustment of Menchero, Wang, and Orr (2012).[2] In contrast to eigenvalue adjustment, bootstrapping historical returns and linear shrinkage do not directly address the SOR bias in portfolio construction. Still, both methods seek to mitigate the negative effects of estimation risk. Given that the SOR bias is a measure of the estimation risk as well, we expect these methods to positively influence that dimension. In our empirical study we find that linear shrinkage only marginally helps to reduce the SOR bias. The simple bootstrapping method hardly mitigates the SOR bias across the analyzed strategies.

The rest of this paper is organized as follows. Section 2 outlines the theoretical framework and provides closed-form solutions for the SOR bias of alternative risk parity strategies. Section 3 provides empirical evidence of the effects of SOR on the realized volatility of portfolio construction strategies and sheds light on the performance of well-known estimation risk-mitigation methodologies on the SOR bias. Section 4 concludes the paper.

2 Second-Order Risk and Risk-Based Portfolio Construction

2.1 Second-Order Risk

In this section we describe the framework we use to assess the estimation risk in portfolio construction strategies. Due to the difficulties in predicting returns, our study focuses on the effects of estimation risk for a set of risk-based portfolio construction strategies which rely on the sample

[2]Among others, Johnstone (2001), Ledoit and Wolf (2004b), Karoui (2008), and Stefanovits, Schubiger, and Wütrich (2015) study the spectrum of the sample covariance matrix and propose alternative estimators of the covariance matrix, derived by individual adjustment of each eigenvalue. Although the sample estimate of the covariance matrix is an unbiased estimator of the true covariance matrix, these authors demonstrate that its eigenvalues greatly deviate from the true ones, especially when the number of assets considered greatly exceed the sample size. There is a systematic upward bias for the largest sample eigenvalues, whereas the smallest ones appear to be slightly biased downwards.

covariance matrix of the assets' excess returns as the sole parameter. We consider a sample period of length $T \in \mathbb{N}$, (e.g., T trading days) and a realized (or out-of-sample) period of the same length. Going forward we will differentiate between statistics constructed for the in-sample and those constructed for the realized period by labeling them accordingly. In this setup, we consider a universe of N assets with sample excess returns $\hat{\mathbf{R}} \in \mathbb{R}^{N\times T}$ and covariance matrix $\hat{\boldsymbol{\Omega}} \in \mathbb{R}^{N\times N}$, where

$$\hat{\boldsymbol{\Omega}} = \frac{1}{T}\hat{\mathbf{R}}\hat{\mathbf{R}}'. \tag{1}$$

Analogously, the realized excess return is denoted by $\mathbf{R} \in \mathbb{R}^{N\times T}$ and the covariance matrix by $\boldsymbol{\Omega} \in \mathbb{R}^{N\times N}$, where

$$\boldsymbol{\Omega} = \frac{1}{T}\mathbf{R}\mathbf{R}'. \tag{2}$$

Then, we define $\hat{\mathbf{w}} := w(\hat{\mathbf{R}}, \hat{\boldsymbol{\Omega}})$ to be a vector of portfolio weights derived on the basis of sample returns and volatility matrix. In our framework, the estimation risk is measured by looking at the second-order risk of portfolio strategies, defined as the ratio of realized over sample portfolio variance. Hence, for a portfolio strategy $\hat{\mathbf{w}}$, the corresponding second-order risk bias can be computed as follows

$$SOR(\hat{\mathbf{w}}) := \frac{\sigma^2_{realized}(\hat{\mathbf{w}})}{\sigma^2_{sample}(\hat{\mathbf{w}})} = \frac{\mathbb{E}\left[\hat{\mathbf{w}}'\boldsymbol{\Omega}\hat{\mathbf{w}}\right]}{\mathbb{E}\left[\hat{\mathbf{w}}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}\right]}. \tag{3}$$

On the one hand, an SOR bias equal to 1 characterizes a robust portfolio construction strategy. This is trivially the case for the $1/N$ portfolio, whose weights are proportional to

$$\mathbf{w}_{1/N} := \mathbf{1}. \tag{4}$$

Obviously, these weights do not depend on the sample covariance matrix. The SOR bias for this portfolio is derived as follows

$$SOR(\mathbf{w}_{1/N}) = \frac{\mathbb{E}\left[\mathbf{1}'\boldsymbol{\Omega}\mathbf{1}\right]}{\mathbb{E}\left[\mathbf{1}'\hat{\boldsymbol{\Omega}}\mathbf{1}\right]} = \frac{\mathbf{1}'\boldsymbol{\Omega}\mathbf{1}}{\mathbf{1}'\mathbb{E}\left[\hat{\boldsymbol{\Omega}}\right]\mathbf{1}} = 1. \tag{5}$$

On the other hand, an SOR bias different from 1 indicates that the portfolio strategy systematically suffers from estimation risk; this calls for the application of estimation risk mitigation techniques.

Another popular example is the minimum-variance portfolio (MV). Shepard (2009) shows that the MV portfolio with weights proportional to

$$\hat{\mathbf{w}}_{MV} := \hat{\mathbf{\Omega}}^{-1}\mathbf{1}, \tag{6}$$

suffers from a systematic SOR bias, which is proportional to the ratio of the number of assets N to the length of the sample period T. More precisely, under the assumption that the underlying asset excess returns are normally distributed with mean zero and covariance matrix $\mathbf{\Omega}$, the sample covariance matrix $\hat{\mathbf{\Omega}}$, as defined in equation (1), follows a Wishart distribution, i.e., $T \times \hat{\mathbf{\Omega}} \sim \mathcal{W}_N(\mathbf{\Omega}, T)$.[3] Using this result, (Shepard 2009) derives a closed-form formula for the SOR bias of the optimal MV portfolio $\hat{\mathbf{w}}_{MV}$ given by equation (6) as

$$\begin{aligned}
SOR(\hat{\mathbf{w}}_{MV}) &= \frac{\mathbb{E}\left[\hat{\mathbf{w}}_{MV}'\mathbf{\Omega}\hat{\mathbf{w}}_{MV}\right]}{\mathbb{E}\left[\hat{\mathbf{w}}_{MV}'\hat{\mathbf{\Omega}}\hat{\mathbf{w}}_{MV}\right]} = \frac{\mathbf{1}'\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1}\right]\mathbf{1}}{\mathbf{1}'\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1}\hat{\mathbf{\Omega}}\hat{\mathbf{\Omega}}^{-1}\right]\mathbf{1}} \\
&\overset{(A)}{\simeq} \left(1 - \frac{N}{T}\right)^{-3} \frac{\mathbf{1}'\mathbf{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1}\right]\mathbf{1}} \\
&\overset{(B)}{\simeq} \left(1 - \frac{N}{T}\right)^{-2} \frac{\mathbf{1}'\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1}\right]\mathbf{1}}{\mathbf{1}'\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1}\right]\mathbf{1}} \\
&= \left(1 - \frac{N}{T}\right)^{-2},
\end{aligned}$$

where approximations (A) and (B) are based on results regarding the first two moments of the inverse Wishart distribution which drop $\mathcal{O}(1/N)$ and $\mathcal{O}(1/T)$ terms for simplicity.[4]

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[3] The Wishart distribution is the multivariate case of the χ^2-distribution.

[4] In particular, for the Inverse Wishart distribution, one has

$$\begin{aligned}
(A) \qquad & \mathbb{E}\left[\hat{\Omega}^{-1}\Omega\hat{\Omega}^{-1}\right] = \frac{(T-1)T^2}{(T-N)(T-N-1)(T-N-3)}\Omega^{-1} \simeq \left(1 - \frac{N}{T}\right)^{-3}\Omega^{-1} \overset{(B)}{\simeq} \left(1 - \frac{N}{T}\right)^{-2}\mathbb{E}\left[\hat{\Omega}^{-1}\right] \\
(B) \qquad & \mathbb{E}\left[\hat{\Omega}^{-1}\right] = \left(1 - \frac{N-1}{T}\right)^{-1}\Omega^{-1} \simeq \left(1 - \frac{N}{T}\right)^{-1}\Omega^{-1}.
\end{aligned}$$

Not only does Shepard (2009) quantify the second-order risk induced by the weights of the MV portfolio, he also uses the above result to obtain an unbiased estimator for its variance:

$$\hat{\sigma}^2_{MV} := \hat{\mathbf{w}}'_{MV} \hat{\mathbf{\Omega}} \hat{\mathbf{w}}_{MV} \left(1 - \frac{N}{T}\right)^{-2}, \tag{7}$$

and it thus holds that

$$\mathbb{E}\left[\hat{\sigma}^2_{MV} \middle| \mathbf{\Omega}\right] = \hat{\sigma}^2_{MV}. \tag{8}$$

This correction depends only on the number of assets and the number of observations used for the in-sample calculation of the covariance matrix. For example, assuming an in-sample period of 1 year (with approximately 252 trading days) and an asset universe of 75 securities, equation (7) implies that the in-sample predicted variance of the MV portfolio returns is twice as high as the realized out-of-sample portfolio variance.

2.2 Second-Order Risk of Alternative Risk Parity Strategies

In the following we examine the SOR bias pertaining to alternative risk parity strategies that invest in uncorrelated risk sources embedded in the underlying asset universe. These uncorrelated risk sources are the principal components (or principal portfolios, PP) of the PCA decomposition of the sample covariance matrix, i.e.,

$$\hat{\mathbf{\Omega}} = \hat{\mathbf{U}}' \hat{\mathbf{\Lambda}} \hat{\mathbf{U}}, \tag{9}$$

where $\hat{\mathbf{U}}$ is the matrix of eigenvectors of $\hat{\mathbf{\Omega}}$ representing the loadings of the principal components and $\hat{\mathbf{\Lambda}} = \text{diag}(\lambda_i)_{i=1,\ldots,N}$ is the diagonal matrix containing the variances of the corresponding eigenvalues. Because of the uncorrelatedness of the PPs, their marginal contribution to portfolio diversification is considerable. The overall portfolio variance σ^2_{sample} can be represented as the weighted sum of the variances of the PPs:

$$\sigma^2_{sample} := \hat{\mathbf{w}}' \hat{\mathbf{\Omega}} \hat{\mathbf{w}} = \sum_{i=1}^{N} \tilde{w}_i^2 \hat{\lambda}_i, \tag{10}$$

where $\tilde{\mathbf{w}} := \hat{\mathbf{U}}\hat{\mathbf{w}}$ translates portfolio weights $\hat{\mathbf{w}}$ into principal portfolio weights $\tilde{\mathbf{w}}$. Then, a portfolio's SOR bias is the weighted average (based on the weights of the PP) of the SOR bias of each single PP. Systematically altering the weighting of the PPs can thus be expected to lead to a systematic change in the SOR bias of the portfolio. We investigate this aspect by looking at the related portfolio strategies.

2.2.1 Inverse-variance in principal portfolios

First, we consider the $1/V$ portfolio that invests in each PP proportionally to the inverse of the corresponding PP's variance[5] and thus strongly loads on low volatility PPs. The portfolio weights are proportional to

$$\hat{\mathbf{w}}_{1/V} := \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}^{-1}\mathbf{1}. \tag{11}$$

To compute the SOR bias we consider the unnormalized version of portfolio weights in equation (11) and separately evaluate the realized and the sample risk estimates. For the former, we obtain

$$\begin{aligned}
\mathbb{E}\left[\hat{\mathbf{w}}_{1/V}'\mathbf{\Omega}\hat{\mathbf{w}}_{1/V}\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbf{1}'\hat{\mathbf{\Lambda}}^{-1}\hat{\mathbf{U}}'\mathbf{\Omega}\hat{\mathbf{U}}\hat{\mathbf{\Lambda}}^{-1}\mathbf{1}\middle|\hat{\mathbf{U}}\right]\right] \\
&= \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\mathbb{E}\left[\hat{\mathbf{U}}\hat{\mathbf{\Lambda}}^{-1}\hat{\mathbf{U}}'\mathbf{\Omega}\hat{\mathbf{U}}\hat{\mathbf{\Lambda}}^{-1}\hat{\mathbf{U}}'\middle|\hat{\mathbf{U}}\right]\hat{\mathbf{U}}\mathbf{1}\right] \\
&= \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1}\middle|\hat{\mathbf{U}}\right]\hat{\mathbf{U}}\mathbf{1}\right] \\
&\overset{(A)}{\simeq} \left(1-\frac{N}{T}\right)^{-2}\mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\hat{\mathbf{\Omega}}^{-1}\hat{\mathbf{U}}\mathbf{1}\right] \\
&= \left(1-\frac{N}{T}\right)^{-2}\sum_{k=1}^{N}\mathbb{E}\left[1/\hat{\lambda}_k\right].
\end{aligned}$$

For the sample risk estimate, we obtain

$$\mathbb{E}\left[\hat{\mathbf{w}}_{1/V}'\hat{\mathbf{\Omega}}\hat{\mathbf{w}}_{1/V}\right] = \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{\Lambda}}^{-1}\hat{\mathbf{U}}'\hat{\mathbf{\Omega}}\hat{\mathbf{U}}\hat{\mathbf{\Lambda}}^{-1}\mathbf{1}\right] = \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{\Lambda}}^{-1}\hat{\mathbf{\Lambda}}\hat{\mathbf{\Lambda}}^{-1}\mathbf{1}\right] = \sum_{k=1}^{N}\mathbb{E}\left[1/\hat{\lambda}_k\right], \tag{12}$$

[5] As every one of the principal portfolios can be bought or sold, there exist 2^N asset allocations, where N is the number of principal portfolios, all of which are inverse variance strategies. A unique strategy is consequently obtained by imposing a sign constraint on the principal portfolios. For the empirical part of this paper, we align the sign of each principal portfolio with the one of its corresponding historical risk premia over a given time period.

Hence, the SOR bias of the $1/V$ portfolio can be approximated as follows:

$$SOR(\hat{\mathbf{w}}_{1/V}) = \frac{\mathbb{E}\left[\hat{\mathbf{w}}_{1/V}^{'}\boldsymbol{\Omega}\hat{\mathbf{w}}_{1/V}\right]}{\mathbb{E}\left[\hat{\mathbf{w}}_{1/V}^{'}\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{1/V}\right]} \simeq \left(1 - \frac{N}{T}\right)^{-2}, \tag{13}$$

The SOR bias derived in equation (13) for the $1/V$ strategy is thus of the same magnitude as the SOR bias of the MV portfolio in equation (7). Thus, equation (7) is an unbiased estimator of the variance of the $1/V$ strategy as well.

2.2.2 Diversified Risk Parity: Inverse volatility along principal portfolios

Second, we consider the diversified risk parity (DRP) strategy. Similar to the $1/V$ portfolio, the DRP strategy invests in uncorrelated risk sources as provided by the PPs pertaining to the PCA decomposition of the sample covariance matrix. The DRP strategy especially leverages the following diversification measure of Meucci (2009):

$$\mathcal{N}_{Ent}(\mathbf{w}) = \exp\left(-\sum_{i=1}^{N} p(\tilde{w}_i)\ln p(\tilde{w}_i)\right), \tag{14}$$

where

$$p(\tilde{w}_i) = \frac{\tilde{w}_i^2\hat{\lambda}_i}{\sum_{i=1}^{N}\tilde{w}_i^2\hat{\lambda}_i}, \qquad i = 1, ..., N. \tag{15}$$

$\mathcal{N}_{Ent}(\mathbf{w})$ can be interpreted as the number of uncorrelated risk sources that a given portfolio strategy $\mathbf{w}$ is investing in. One has $\mathcal{N}_{Ent}(\mathbf{w}) = 1$ for a fully concentrated strategy and $\mathcal{N}_{Ent}(\mathbf{w}) = N$ for a fully diversified strategy. The weights of the DRP strategy are constructed by maximizing the diversification measure $\mathcal{N}_{Ent}(\mathbf{w})$, i.e.,

$$\hat{\mathbf{w}}_{DRP} = \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}}\,\mathcal{N}_{Ent}(\mathbf{w}). \tag{16}$$

Maximizing the diversification is equivalent to allocating an equal risk budget to every uncorrelated PP, subject to a set of allocation constraints $\mathcal{C}$. In the absence of constraints, the DRP strategy has a closed-form solution that prescribes inverse volatility investing along principal portfolios.[6] Its weights are proportional to

$$\hat{\mathbf{w}}_{DRP} := \hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}^{\mathbf{-1/2}}\mathbf{1}. \tag{17}$$

By weighting each PP inversely to the square root of its variance, the DRP ensures an equal contribution to the total variance by each PP, reflecting the underlying idea of maximum diversification. The DRP weighting appears to be more moderate, when compared to that of the $1/V$ strategy. This more moderate weighting of the PPs should translate into a lower SOR bias. Similar to the derivation of the SOR bias for the $1/V$ portfolio, we calculate the realized risk as well as the sample risk estimates separately. For the realized risk, we have

$$\begin{aligned}
\mathbb{E}\left[\hat{\mathbf{w}}_{DRP}'\boldsymbol{\Omega}\hat{\mathbf{w}}_{DRP}\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbf{1}'\hat{\boldsymbol{\Lambda}}^{-1/2}\hat{\mathbf{U}}'\boldsymbol{\Omega}\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}^{-1/2}\mathbf{1}\middle|\hat{\mathbf{U}}\right]\right] \\
&= \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\mathbb{E}\left[\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}^{-1/2}\hat{\mathbf{U}}'\boldsymbol{\Omega}\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}^{-1/2}\hat{\mathbf{U}}'\middle|\hat{\mathbf{U}}\right]\hat{\mathbf{U}}\mathbf{1}\right] \\
&= \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\mathbb{E}\left[\hat{\boldsymbol{\Omega}}^{-1/2}\boldsymbol{\Omega}\hat{\boldsymbol{\Omega}}^{-1/2}\middle|\hat{\mathbf{U}}\right]\hat{\mathbf{U}}\mathbf{1}\right] \\
&\overset{(C)}{\simeq} \left(1-\frac{N}{T}\right)^{-1}\mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\hat{\mathbf{U}}\mathbf{1}\right] = \left(1-\frac{N}{T}\right)^{-1} N,
\end{aligned}$$

where equation (C) follows from a heuristic derivation. The expression

$$\mathbb{E}\left[\hat{\boldsymbol{\Omega}}^{-1/2}\boldsymbol{\Omega}\hat{\boldsymbol{\Omega}}^{-1/2}\middle|\hat{\mathbf{U}}\right],$$

dubbed the "Bias Matrix," cannot be derived analytically. Thus, we verify the validity of our approximation (C) via a Monte Carlo simulation, which we outline in Appendix 5.

The sample variance estimate can be calculated as follows:

$$\begin{aligned}
\mathbb{E}\left[\hat{\mathbf{w}}_{DRP}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{DRP}\right] &= \mathbb{E}\left[\mathbf{1}'\hat{\boldsymbol{\Lambda}}^{-1/2}\hat{\mathbf{U}}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}^{\mathbf{-1/2}}\mathbf{1}\right] \\
&= \mathbb{E}\left[\mathbf{1}'\hat{\boldsymbol{\Lambda}}^{-1/2}\hat{\mathbf{U}}'\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}\hat{\mathbf{U}}'\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}^{\mathbf{-1/2}}\mathbf{1}\right] \\
&= \mathbb{E}\left[1'\hat{\boldsymbol{\Lambda}}^{-1/2}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{\Lambda}}^{-1/2}1\right] = N.
\end{aligned}$$

[6]As with the $1/V$ strategy, uniqueness of the asset allocation is again guaranteed by means of sign constraints based on the historically observed risk premia.

The SOR bias for the DRP can be approximated as follows:

$$SOR(\hat{w}_{DRP}) := \frac{\mathbb{E}\left[\hat{\mathbf{w}}_{DRP}^{'}\boldsymbol{\Omega}\hat{\mathbf{w}}_{DRP}\right]}{\mathbb{E}\left[\hat{\mathbf{w}}_{DRP}^{'}\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{DRP}\right]} \overset{(C)}{\simeq} \left(1 - \frac{N}{T}\right)^{-1}. \tag{18}$$

Analogously to equation (7), we can derive an unbiased estimator of the DRP portfolio's variance:

$$\hat{\sigma}_{DRP}^2 := \hat{\mathbf{w}}_{DRP}^{'}\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{DRP}\left(1 - \frac{N}{T}\right)^{-1}. \tag{19}$$

As in the cases of the MV and $1/V$ portfolios, this correction depends only on the number of assets and the number of observations used for the in-sample calculation of the covariance matrix. For example, assuming an in-sample period of 1 year (with approximately 252 trading days) and an asset universe of 125 securities, equation (7) implies that the in-sample predicted portfolio variance is twice as high as the realized out-of-sample portfolio variance.

2.2.3 Principal Risk Parity: Equally-weighted principal portfolios

In light of the above results, we present an alternative risk parity strategy, referred to as principal risk parity (PRP). Similar to the $1/V$ and DRP portfolios, the PRP strategy invests in uncorrelated risk sources as given by the PPs, but in a more natural fashion. The PRP portfolio budgets the risk proportionally to each PP's contribution to the total variance. As a result, the lion's share of the capital is allocated to the most significant principal portfolios, thus getting around the less significant PPs. Technically, we are simply assigning equal weights to the PPs, prompting us to call this strategy the principal risk parity strategy.[7] To obtain the strategy weights $\hat{\mathbf{w}}_{PRP}$, we need to solve

$$\hat{\mathbf{w}}_{PRP} = \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}}\, \mathcal{M}_{Ent}(\mathbf{w}), \tag{20}$$

[7] As with the $1/V$ and DRP strategies, uniqueness of the asset allocation is guaranteed by means of sign constraints based on the historically observed risk premia of the PPs.

where $\mathcal{M}_{Ent}$ is defined as

$$\mathcal{M}_{Ent}(\mathbf{w}) = \exp\left(-\sum_{i=1}^{N} q(\tilde{w}_i)\ln q(\tilde{w}_i)\right), \tag{21}$$

and

$$q(\tilde{w}_i) = \frac{\tilde{w}_i^2}{\sum_{i=1}^{N} \hat{w}_i^2}, \qquad i = 1, ..., N. \tag{22}$$

The unconstrained version has a closed-form solution, proportional to

$$\hat{\mathbf{w}}_{PRP} := \hat{\mathbf{U}}\mathbf{1}. \tag{23}$$

Intuition suggest that the SOR bias should be lower for the PRP than for the $1/V$, MV, and DRP portfolios, because PRP allocates away from low volatility PPs. Our calculations for PRP confirm this intuition. For the realized variance of the PRP portfolio, we obtain

$$\mathbb{E}\left[\hat{\mathbf{w}}_{PRP}'\boldsymbol{\Omega}\hat{\mathbf{w}}_{PRP}\right] = \mathbb{E}\left[\mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\boldsymbol{\Omega}\hat{\mathbf{U}}\mathbf{1}\middle|\hat{\mathbf{U}}\right]\right] \overset{(A)}{=} \mathbb{E}\left[\mathbf{1}'\mathbb{E}\left[\hat{\mathbf{U}}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{U}}\middle|\hat{\mathbf{U}}\right]1\right] = \mathbb{E}\left[\mathbf{1}'\hat{\boldsymbol{\Lambda}}\mathbf{1}\right], \tag{24}$$

where equation (A) relies on the fact that, conditional on $\hat{\mathbf{U}}$:

$$T \times \hat{\boldsymbol{\Omega}} \sim \mathcal{W}_N(\boldsymbol{\Omega}, T) \Rightarrow T \times \hat{\mathbf{U}}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{U}}' \sim \mathcal{W}_N\left(\hat{\mathbf{U}}\boldsymbol{\Omega}\hat{\mathbf{U}}', T\right). \tag{25}$$

Thus,

$$\mathbb{E}\left[\hat{\mathbf{U}}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{U}}\middle|\hat{\mathbf{U}}\right] = \hat{\mathbf{U}}'\boldsymbol{\Omega}\hat{\mathbf{U}}. \tag{26}$$

The sample variance can be calculated as

$$\mathbb{E}\left[\hat{\mathbf{w}}_{PRP}'\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{PRP}\right] = \mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\hat{\mathbf{U}}\hat{\boldsymbol{\Lambda}}\hat{\mathbf{U}}'\hat{\mathbf{U}}\mathbf{1}\right] = \mathbb{E}\left[\mathbf{1}'\hat{\boldsymbol{\Lambda}}\mathbf{1}\right]. \tag{27}$$

Hence, the SOR bias of the PRP portfolio is

$$SOR(\hat{\mathbf{w}}_{PRP}) = \frac{\mathbb{E}\left[\hat{\mathbf{w}}_{PRP}^{'}\boldsymbol{\Omega}\hat{\mathbf{w}}_{PRP}\right]}{\mathbb{E}\left[\hat{\mathbf{w}}_{PRP}^{'}\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{PRP}\right]} = 1. \tag{28}$$

In contrast to equation (7) for the MV portfolio, and equation (19) for the DRP portfolio, the sample variance of the PRP strategy can be considered to be an unbiased estimator of its true variance, conditional on correctly estimating the correlation structure of the investment universe:

$$\hat{\sigma}_{PRP}^2 := \hat{\mathbf{w}}_{PRP}^{'}\hat{\boldsymbol{\Omega}}\hat{\mathbf{w}}_{PRP}. \tag{29}$$

2.3 Mitigating estimation risk

2.3.1 Eigenvalue adjustment

Not knowing the exact size of the SOR bias of a given portfolio strategy, Menchero, Wang, and Orr (2012) developed an SOR-unbiased estimator of the covariance matrix. Their method is based on the observation that each of the principal portfolios suffers from an SOR bias. In particular, the SOR bias of a given PP is higher if its associated eigenvalue is smaller, see Table 1, Panel B.

Their method aims at eliminating the SOR bias of a given portfolio by correcting the SOR bias of the underlying PPs. In this regard, one simulates asset returns that follow a joint normal distribution with mean zero according to the sample covariance matrix. For each simulation $s = 1, .., S$, the authors compute the variances $\lambda_{k,s}$ of the PPs for $k = 1, .., N$ via a PCA of the covariance matrix $\boldsymbol{\Omega}_{\mathbf{s}}$

$$\boldsymbol{\Lambda}_{\mathbf{s}} := \mathbf{U}_s^{'}\boldsymbol{\Omega}_s\mathbf{U}_s. \tag{30}$$

These variances are put into relation with the diagonal elements of a matrix $\tilde{\boldsymbol{\Lambda}}_s$, which is obtained assuming the sample covariance matrix is the true covariance matrix of the assets' returns, i.e.,

$$\tilde{\boldsymbol{\Lambda}}_{\mathbf{s}} = \mathbf{U}_s^{'}\hat{\boldsymbol{\Omega}}\mathbf{U}_s. \tag{31}$$

Table 1
Descriptive Statistics of the Fama French Industry Portfolios

The table lists the descriptive statistics for the 10 industry portfolios from CRSP (Center for Research in Security Prices). Industry portfolios are constructed by equally weighting a selection of US equities grouped by industry type according to the corresponding SIC codes (Standard Industrial Classification). Industry sectors are caracterized by a short name. This is reported together with a short description. For more details please visit the website: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*. Panel A gives performance and risk statistics of each industry portfolio. Annualized average return and volatility figures are reported together with the corresponding Sharpe Ratio. Value at risk and expected shortfall are computed at the 95% confidence interval over a 1 year period. Maximum drawdown is calculated over a one year period as well. Second order risk (SOR) bias is additionally reported. Panel B reports the same statistics for the principal portfolios' (PPs), result of a PCA decomposition of asset universe composed by the 10 industry portfolios. Results are derived from 10 years of daily returns ranging from April 2007 to March 2017.

Short Name	Description	Return	Volatility	Sharpe Ratio	Value-at-Risk (at 95% level)	Expected Shortfall (at 95% level)	Maximum Drawdown	SOR Bias
Panel A: Industry Portfolios								
NoDur	Consumer Non-Durables	0.7%	4.3%	0.16	-6.4%	-8.2%	-33.2%	1.06
Durbl	Consumer Durables	0.6%	5.9%	0.10	-9.2%	-11.6%	-42.9%	1.06
Manuf	Manufacturing	0.7%	5.6%	0.12	-8.5%	-10.9%	-39.8%	1.07
Enrgy	Oil, Gas, and Coal Extraction and Products	0.1%	8.7%	0.02	-14.2%	-17.9%	-57.7%	1.06
HiTec	Business Equipment	0.7%	4.7%	0.15	-7.1%	-9.0%	-31.4%	1.06
Telcm	Telephone and Television Transmission	0.7%	5.4%	0.12	-8.2%	-10.5%	-38.4%	1.06
Shops	Wholesale, Retail, and Some Services	0.5%	4.9%	0.11	-7.6%	-9.6%	-37.2%	1.06
Hlth	Healthcare, Medical Equipment, and Drugs	0.8%	4.8%	0.17	-7.1%	-9.1%	-27.8%	1.08
Utils	Utilities	0.5%	4.1%	0.13	-6.2%	-7.9%	-27.0%	1.05
Other	Other	0.6%	4.5%	0.14	-6.8%	-8.7%	-29.6%	1.07
Panel B: Principal Portfolios								
PP1	Principal Portfolio 1	1.7%	15.9%	0.11	-24.5%	-31.1%	-110.7%	1.04
PP2	Principal Portfolio 2	0.1%	4.7%	0.02	-7.8%	-9.7%	-23.1%	1.01
PP3	Principal Portfolio 3	0.0%	2.5%	0.02	-4.0%	-5.1%	-10.3%	0.95
PP4	Principal Portfolio 4	0.3%	2.2%	0.12	-3.8%	-4.8%	-9.4%	1.07
PP5	Principal Portfolio 5	0.1%	1.8%	0.06	-3.1%	-3.8%	-7.9%	1.16
PP6	Principal Portfolio 6	0.0%	1.6%	0.03	-2.6%	-3.3%	-6.3%	1.27
PP7	Principal Portfolio 7	0.2%	1.4%	0.11	-2.2%	-2.8%	-6.8%	1.40
PP8	Principal Portfolio 8	0.0%	1.3%	0.03	-2.1%	-2.6%	-6.5%	1.63
PP9	Principal Portfolio 9	0.0%	1.2%	0.04	-2.0%	-2.5%	-4.2%	1.99
PP10	Principal Portfolio 10	0.1%	1.2%	0.05	-2.0%	-2.4%	-5.5%	2.73

Averaging over all simulations gives a simulated SOR bias β_k for every single PP k,

$$\beta_k := \frac{1}{S}\sum_{s=1}^{S}\sqrt{\frac{\tilde{\lambda}_{k,s}}{\lambda_{k,s}}}. \tag{32}$$

For every PP, the sample variance (or eigenvalue) is multiplied by the square of β_k, to build the diagonal elements $\lambda_k^{eig-adj}$, for $k = 1, .., N$ of a matrix $\mathbf{\Lambda}^{eig-adj}$ containing SOR-corrected variances

$$\lambda_k^{eig-adj} := \beta_k^2 \lambda_k. \tag{33}$$

The ensuing matrix $\mathbf{\Lambda}^{eig-adj}$ then replaces the diagonal matrix containing the sample eigenvalues in the PCA decomposition to give an SOR-corrected estimator $\hat{\mathbf{\Omega}}^{eig-adj}$ of the covariance matrix of the assets' returns:

$$\mathbf{\Omega}^{eig-adj} = \hat{\mathbf{U}}\mathbf{\Lambda}^{eig-adj}\hat{\mathbf{U}}'. \tag{34}$$

2.3.2 Bootstrapping

Another simulation-based technique for mitigating estimation risk can be easily obtained by bootstrapping the asset returns instead of their corresponding eigenvectors. Portfolio optimization techniques might be confounded by extreme values in the estimates of the covariance matrix, driven by a low number of observations in the sample period. This problem might be avoided by bootstrapping the asset returns from the sample period. Thus, the simulated covariance matrices $\mathbf{\Omega_s}$, for $s = 1, .., S$, can be used to compute optimal portfolio weights $\mathbf{w}_s = f(\mathbf{\Omega_s})$ and finally—given a sufficiently large number of simulations—the average portfolio weights across simulations might prove to be more robust to estimation risk.

$$\mathbf{w}^{bootstrap} = \frac{1}{S}\sum_{s=1}^{S}\mathbf{w}_s. \tag{35}$$

This simple procedure aims at avoiding extreme and highly volatile asset allocations.

2.3.3 Linear shrinkage

Lastly, a popular technique for mitigating estimation risk is the linear shrinkage estimator of Ledoit and Wolf (2004a). The authors argue that extreme values in the variance–covariance matrix of asset returns might often shift the optimal weights towards corner solutions. However, such portfolios often disappoint out-of-sample. They suggest shrinking the sample covariance matrix of asset returns $\hat{\mathbf{\Omega}}$ towards a simple estimate of the covariance matrix $\bar{\mathbf{\Omega}}$ consisting of one asset variance σ and one asset covariance δ, i.e., $\bar{\omega}_{i,j} = \sigma$, for $i = j$ and $\bar{\omega}_{i,j} = \delta$, for $i \neq j$. In particular, they derive an estimator as a convex combination of the two, i.e.,

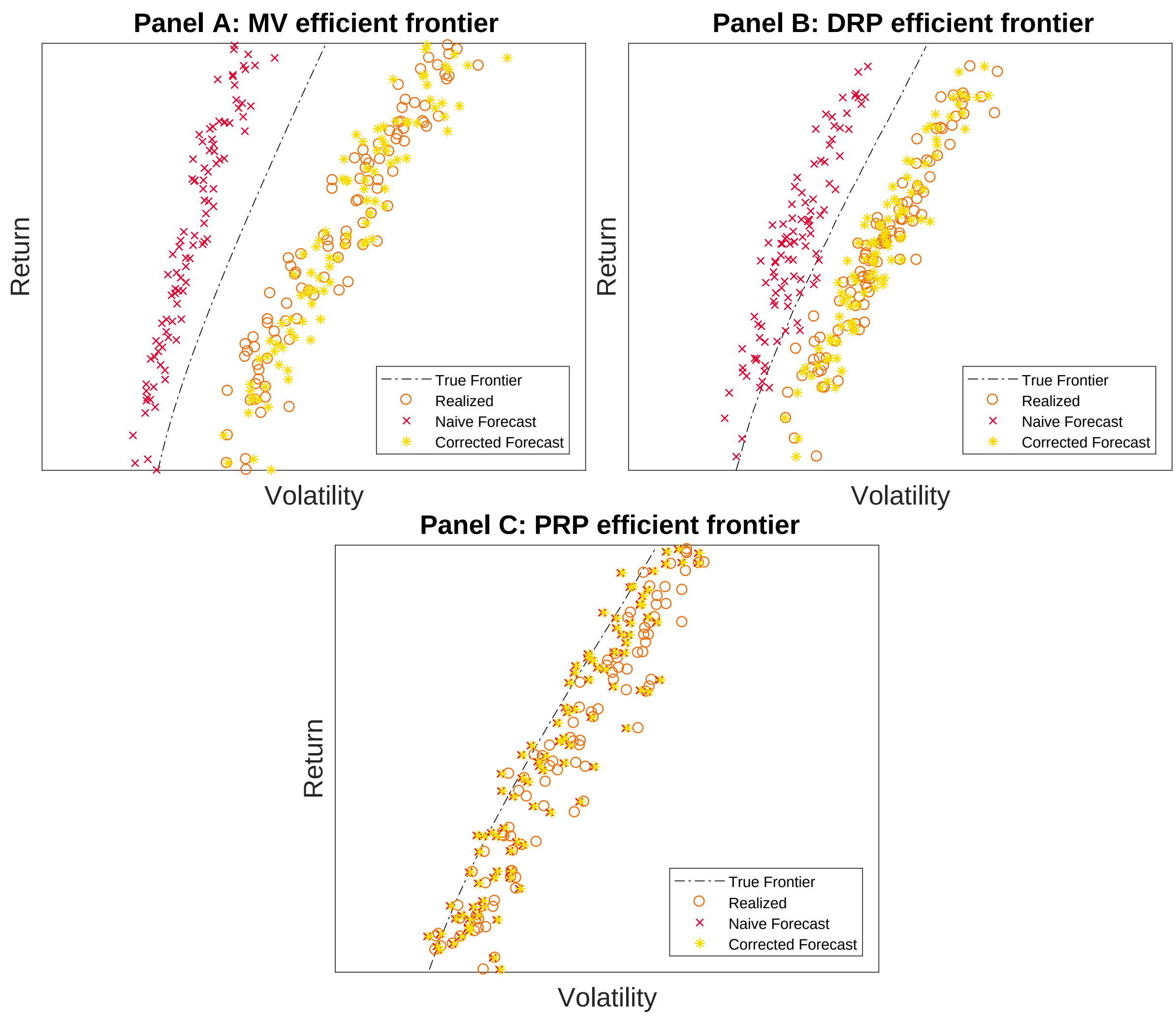
$$\mathbf{\Omega}^{lin-shr} = \tau\bar{\mathbf{\Omega}} + (1-\tau)\hat{\mathbf{\Omega}}, \tag{36}$$

where the weighting factor $0 \leq \tau \leq 1$ is determined via optimization. The use of average values for the variance and covariance components of the simplified covariance matrix, together with the shrinkage procedure, ensures that the linear shrinkage estimator exhibits more moderate values than would have been the case for the sample covariance matrix estimator, thus mitigating the estimation risk.

3 Results

The aim of this section is to illustrate and confirm the results derived in Section 2 based on empirical analyses of simulated as well as real market data. We first start with simulated data, as this allows us to assume a "true" covariance matrix of asset returns. By means of Monte Carlo simulation, we will show the effectiveness of the unbiased estimators of a portfolio's variance in equations (7), (19), and (29). Figure 1 shows the results of the Monte Carlo simulation for the Minimum Variance (MV), Diversified Risk Parity (DRP), and Principal Risk Parity (PRP) portfolios. The results have been obtained by constraining each portfolio to be fully invested and to have fixed expected return R. For each target return R, a new sample covariance matrix $\hat{\mathbf{\Omega}}$ is estimated on 50 daily returns for each of 25 assets, simulated from a multivariate normal distribution with fixed mean and covariance matrix $\boldsymbol{\Omega}$. In order to minimize noise in the simulation, we assume the mean return of each asset to be known, and exclusively focus on the simulation of variances and covariances. For a selection of target returns, various versions of a portfolio's volatility are calculated and compared to each other.

Figure 1. Monte Carlo Simulation: Risk–Return Profile vs. Efficient Frontier
The figure shows the risk return profile of mean variance (MV), diversified risk parity (DRP), and principal risk parity (PRP) portfolios obtained via Monte Carlo simulation. Each panel is dedicated to a single portfolio and shows the true efficient frontier, the naive forecast, the corrected forecast, and the realized risk return profile from each simulation. Results are obtained by simulating 50 daily returns for each of 25 assets, assuming a multivariate normal distribution with known mean and covariance matrix.



In particular, the curve labeled "True Frontier" is calculated assuming perfect knowledge of the covariance matrix, i.e., $\sqrt{\mathbf{w}^{*\prime}\mathbf{\Omega}\mathbf{w}^*}$, where $\mathbf{w}^*$ represents the portfolio that is optimal in terms of the corresponding optimization rule (i.e., either MV, DRP, or PRP) given perfect knowledge about the covariance matrix of asset returns. The dots labeled "Realized" risk represent the actual risk of the portfolio, i.e., $\sqrt{\hat{\mathbf{w}}'\mathbf{\Omega}\hat{\mathbf{w}}}$. The "Naive forecast" is the in-sample estimated volatility of the portfolio, i.e., $\sqrt{\hat{\mathbf{w}}'\hat{\mathbf{\Omega}}\hat{\mathbf{w}}}$. The corrected forecast $\sqrt{\hat{\mathbf{w}}'\hat{\mathbf{\Omega}}\hat{\mathbf{w}}}\left(1-\frac{N}{T}\right)^{-1}$ (and $\sqrt{\hat{\mathbf{w}}'\hat{\mathbf{\Omega}}\hat{\mathbf{w}}}\left(1-\frac{N}{T}\right)^{-1/2}$ respectively) is obtained by correcting the naive forecast according to equation (7) for the MV portfolio and equation (19) for the DRP portfolio), whereas for the PRP portfolio, the naive forecast in equation (29) is already unbiased and does not need to be corrected. Panel A shows how the naive forecast of the MV portfolio's volatility appears to be even better than the true efficient frontier, whereas the realized volatility turns out to be rather higher. The same effect (with a lesser magnitude) can be observed in Panel B for the DRP portfolio, whereas for PRP (see Panel C), the naive, or sample, volatility forecast does not underestimate the realized volatility as expected.

In addition to simulated data, it is of interest to empirically test the results of Section 2 on real market data. The use of real market data for the assessment of second-order risk is not straightforward as it is in the case of simulated data, where the true covariance matrix of the asset returns is known. To facilitate this, we make use of the so called SOR bias statistic to assess the accuracy of a portfolio volatility forecast. This is calculated in the following way. For a set of portfolio weights $\mathbf{w} = (\mathbf{w}_t)_{t=1,...,T}$ the SOR bias statistics for $t = t_0 + 1, ..., T$ is given by

$$B_{t_0+t} := \frac{r_t}{\hat{\sigma}_{t-1}} := \frac{\mathbf{w}'_{t-1}\mathbf{R}^d_t}{\sqrt{\mathbf{w}'_{t-1}\hat{\mathbf{\Omega}}_{t-1}\mathbf{w}_{t-1}}}, \tag{37}$$

where t_0 denotes the length of the in-sample rolling window, $\hat{\mathbf{\Omega}}_{t-1}$ denotes the sample covariance matrix of asset returns, and $\mathbf{w}_{t-1}$ denotes the portfolio weights constructed out of de-meaned in-sample returns $\mathbf{R}^d_{t-t_0}, ..., \mathbf{R}^d_{t-1}$. De-meaned returns are considered for the calculation of the bias statistics. This is done to prevent the measure to suffer from a bias induced by asset returns. As $r_t := \mathbf{w}'_{t-1}\mathbf{R}^d_t$ is the realized de-meaned return of the portfolio at time t, B_{t_0+t} can be considered as a standardized return. Once calculated, the standard deviation of the SOR bias statistic provides a good approximation of the SOR bias of the considered portfolio when looked at in the context of real market data.

As asset universe, we build on the Fama–French industry portfolios as provided by the Center for Research in Security Prices (CRSP).[8] Industry portfolios are constructed by equally weighting a selection of U.S. equities grouped by industry type according to the corresponding SIC codes (Standard Industrial Classification). Table 1 shows a collection of the main statistics for CRSP data composed of daily returns of U.S. equities grouped within ten Industry Portfolios over ten years (from April 2007 to March 2017). These ten portfolios cover the sectors of: Consumer Non-Durables (NoDur), Consumer Durables (Durbl), Manufacturing (Manuf), Oil, Gas, and Coal Extraction and Products (Enrgy), Business Equipment (HiTech), Telephone and Television Transmission (Telcm), Wholesale, Retail and Some Services (Shops), Healthcare, Medical Equipment, and Drugs (Hlth), Utilities (Utils), and Other (Other). The same table also reports statistics for the ten Principal Portfolios, the results of PCA decompositions of the asset universe composed by the ten Industry Portfolios over the ten years considered. From the perspective of the SOR bias, it is interesting to observe how the industry portfolios (constructed by equally weighting a selection of U.S. stocks) exhibit SOR biases close to one (in particular ranging from 1.05 to 1.08), whereas the SOR bias of the Principal Portfolios comes out inversely proportional to their volatility with values close to one for the first three principal portfolios (1.04 for PP1, 1.01 for PP2, and 0.95 for PP3), then increasing to values around 2–3 for the last principal portfolios (1.99 for PP9 and 2.73 for PP10).

Based on this asset universe, Figure 2 reports the SOR bias, calculated as the standard deviation of the bias statistic in equation (37) over the ten-year period (from April 2007 to March 2017) for a set of asset allocation strategies which have been presented in Section 2—$1/N$, MV, $1/V$, DRP, and PRP—together with a random strategy that has been used as an additional benchmark to the $1/N$ portfolio.[9] The displayed sample SOR bias statistics are in line with the SOR biases theoretically derived in Section 2. The same figure reports SOR bias statistics conditional on the

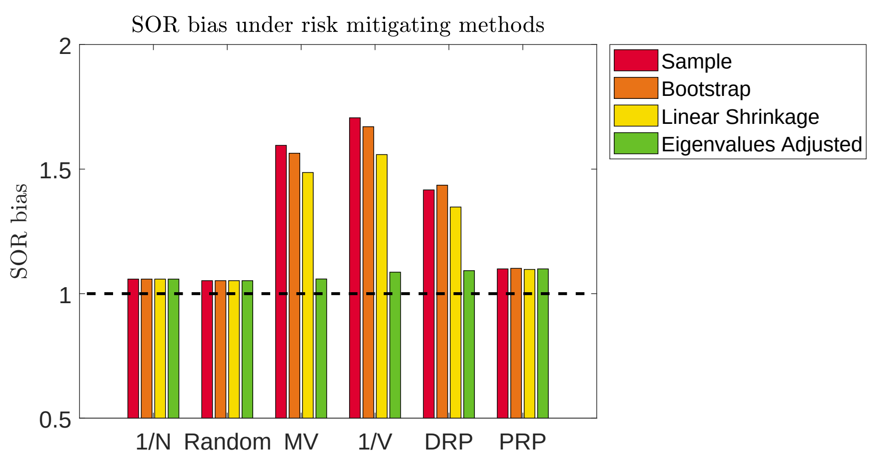
[8] Datasets of from 5 up to 49 industry portfolios are available online. For more details please visit the website: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*.

[9] The uniqueness of the $1/V$, DRP, and PRP strategies is guaranteed by imposing a sign constraint on the PPs. These have to carry a positive as well as historically observed risk premium, measured by a rolling time window of 12 months.

application of risk-mitigating techniques, as described in Section 2. In addition to the SOR bias derived by the portfolio optimization based on the sample co-variance matrix estimate, Figure 2 reports the SOR bias of the selected asset allocation strategies when the optimization routine is run on each of the alternative estimators of the covariance matrix, namely, an estimate based on a simple bootstrapping method, the linear shrinkage, and the eigenvalue-adjusted technique.

Figure 2. Second-Order Risk Bias under Risk Mitigation Methodologies

The figure provides the second-order risk bias under the application of various estimation risk mitigation methodologies for various asset allocation strategies: the equally weighted portfolio ($1/N$), a random portfolio (Rand.), the minimum variance (MV), the inverse variance ($1/V$), the diversified risk parity (DRP), and the principal risk parity portfolio (PRP). The SOR bias is calculated as the standard deviation of the bias statistic in (37) over a ten-year time period, from April 2007 to March 2017. The bias statistic is calculated as of every day based on a rolling window of 30 daily returns. The asset universe considered consists in the ten industry portfolios provided by the CRSP (Center for Research in Security Prices) and described in Table 1. The SOR bias is provided without (Sample) and with the application of an estimation risk mitigation methodology: either the bootstrap, the linear shrinkage, or the eigenvalue adjusting methodology, which were described in Section 2.3.

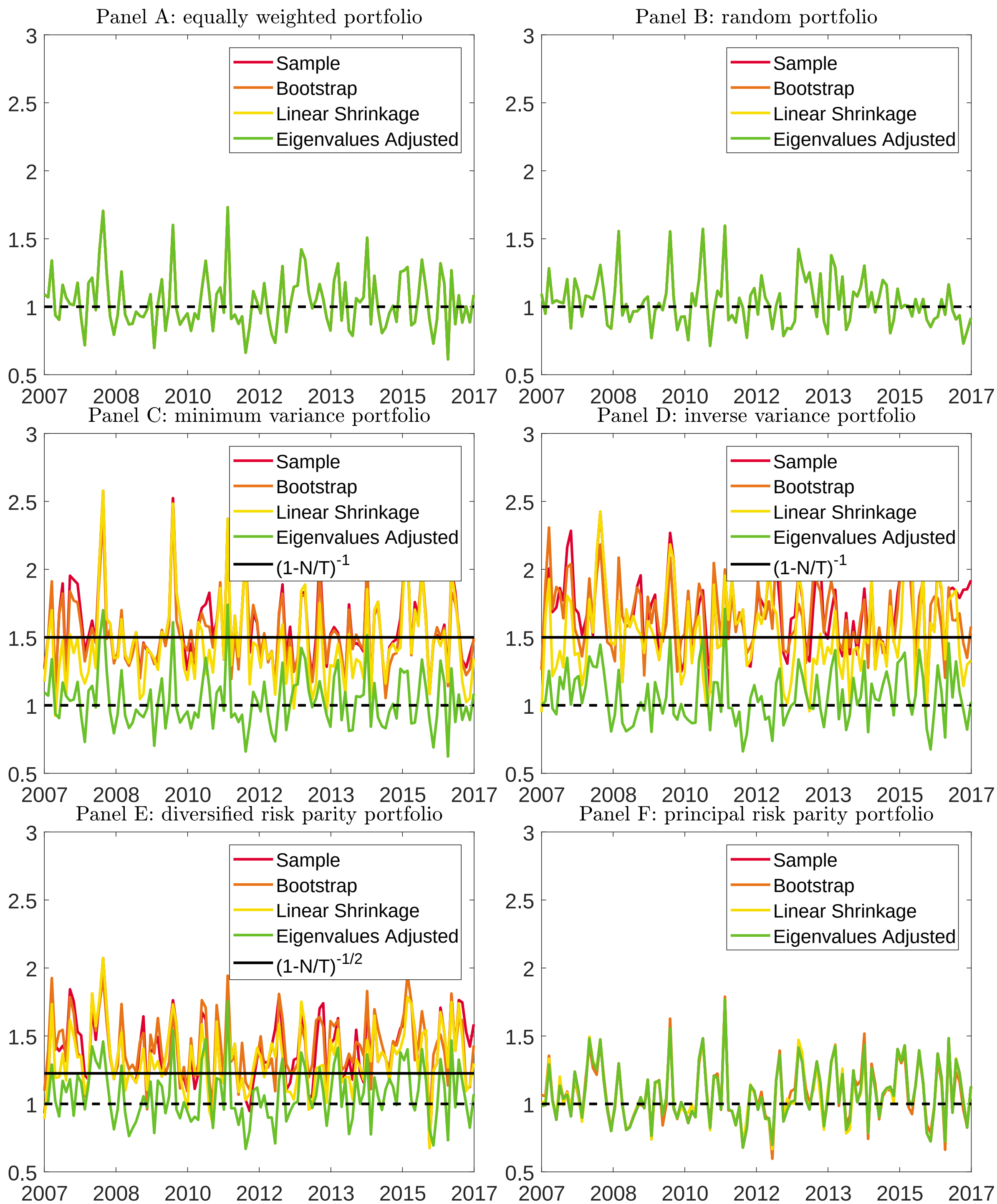


As expected, the equally weighted portfolio and the random strategy do not appear to suffer from an SOR bias: the reported SOR biases, 1.06 for $1/N$ (and 1.05 for the random strategy) are in line with the SOR bias of the single industry portfolios listed in Table 1, and ranging from 1.05 to 1.08, and are mainly induced by the finite time horizon considered and the reliance on real market data rather than simply simulated data. In contrast to these two control strategies, the MV portfolios SOR bias is rather high at 1.60 (sample) and 1.56 (bootstrap)—i.e., the realized volatility of the MV portfolio is about 60% higher than the in-sample volatility. The SOR bias drops significantly to 1.49 under linear shrinkage and even more, to 1.06, under the eigenvalue-adjustment method. This observation is in line with the average SOR biases of the single Industry Portfolios. The $1/V$ portfolio displays SOR biases, slightly higher but similar in magnitude to those of the MV portfolio. The $1/V$ portfolio has an SOR bias of 1.71 (sample version) and 1.67 (bootstrapping). The SOR bias drops to 1.56 under linear shrinkage of the sample co-variance matrix and to 1.09 under the eigenvalue adjustment. The DRP strategy is less affected by SOR bias than the MV and $1/V$ strategies. Its SOR bias is approximately the square root of those of the $1/V$ and MV strategies, as derived in Section 2. We especially observe an SOR bias of 1.42 for the sample and 1.44 for the bootstrap strategies. Applying linear shrinkage or the eigenvalue adjustment appear to give rise to SOR biases of 1.35 and 1.09, respectively. The PRP portfolio displays an SOR bias of 1.10 in its sample and eigenvalue-adjusted versions. The application of bootstrapping and linear shrinkage leaves the bias almost unchanged, at 1.10. The observation of SOR bias statistics slightly higher than one, reported for the strategies in their eigenvalue-adjusted versions, is mainly related to the use of a finite sample of real market data.

In addition to the SOR bias over the whole of the considered time period, Figure 3 depicts the SOR bias statistics for the analyzed portfolio construction strategies over time. The ranking of the SOR bias inferred from Figure 2 also applies over time. The SOR biases of $1/N$ and random strategies oscillate around the expected value of 1. Two major deviations from 1 can be observed for the SOR bias of the $1/N$ strategy in Panel A, prior to the two equity crises in 2000 and 2008. Panel C (MV) and Panel D ($1/V$) are quite similar over time. Panel E reports the SOR bias over time for the DRP portfolio, which is less volatile when compared with the SOR bias of MV and $1/V$ strategies. Panel F gives the SOR bias of the PRP portfolio, which fluctuates around 1 with a slight upward bias. Also, it is less sensitive to the application of methods to mitigate estimation risk. Similarly, applying methods to mitigate the estimation risk over time is in line with the effects observed on average.

Figure 3. Second-Order Risk Bias Over Time

The figure shows the second-order risk bias over time for various asset allocation strategies and risk mitigation methodologies (as described in Section 2.3. Each panel represents the SOR bias over time for an individual portfolio strategy both with and without the application of the various risk mitigation methodologies. SOR bias is calculated via a rolling window of 30 daily returns. Portfolios are constructed out of the ten industry portfolios described in Table 1. Data used range from April 2007 to March 2017.



Figures 4 and 5 both illustrate the SOR bias for MV, DRP, and PRP portfolios as a function of the number of in-sample observations T and the number of assets N. The results proposed in Figure 5 are derived based on a fixed number of industry portfolios (ten) by letting the number of observations T vary from 20 up to 60 daily returns, giving T/N ratios of from 2 up to 6.

Figure 4. Correcting Second-Order Risk Bias for Varying T/N Ratios

The figure shows the second-order risk bias (calculated via SOR bias statistic) for the MV portfolio, the DRP portfolio and the PRP portfolio under varying ratios of number of monthly returns (T) considered in-sample over number of industry portfolios (N). The number of industry portfolios is kept constant within each panel. For Panel A, a ratio $T/N = 3$ corresponds to the case of 5 industry portfolios and 15 monthly in-sample observations. Panel B displays the SOR bias constructed out of 10 industry portfolios, Panel C with 30 industry portfolios, and Panel D reports the SOR bias based on 48 industry portfolios. Additionally, for MV and DRP strategies, the corrected SOR bias, using estimators from equations (7), and (19) are reported. Results are derived using the 10 industry portfolios described in Table 1 and analogous industry portfolios (5, 30, and 48) as consistently provided by CRSP (Center for Research in Security Prices). For more details please visit the website: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html*. Results are generated using data ranging from April 2007 to March 2017.

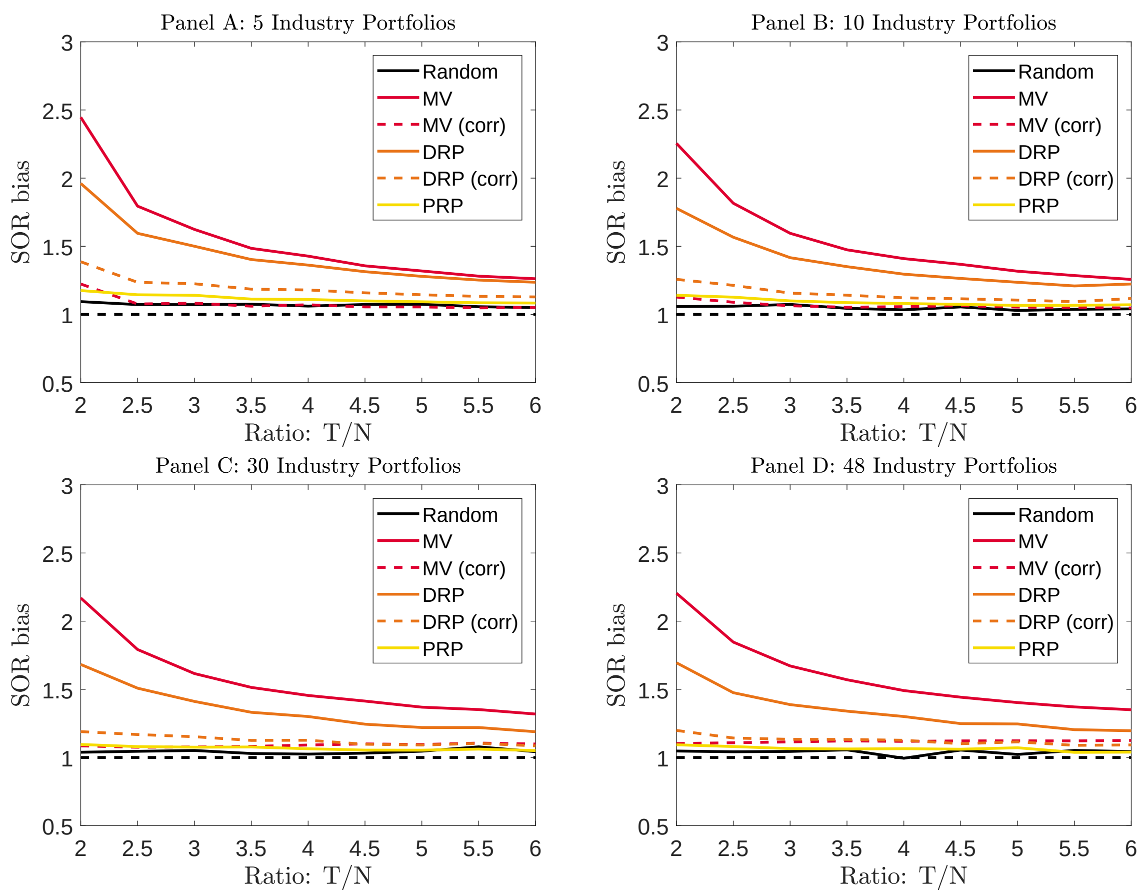
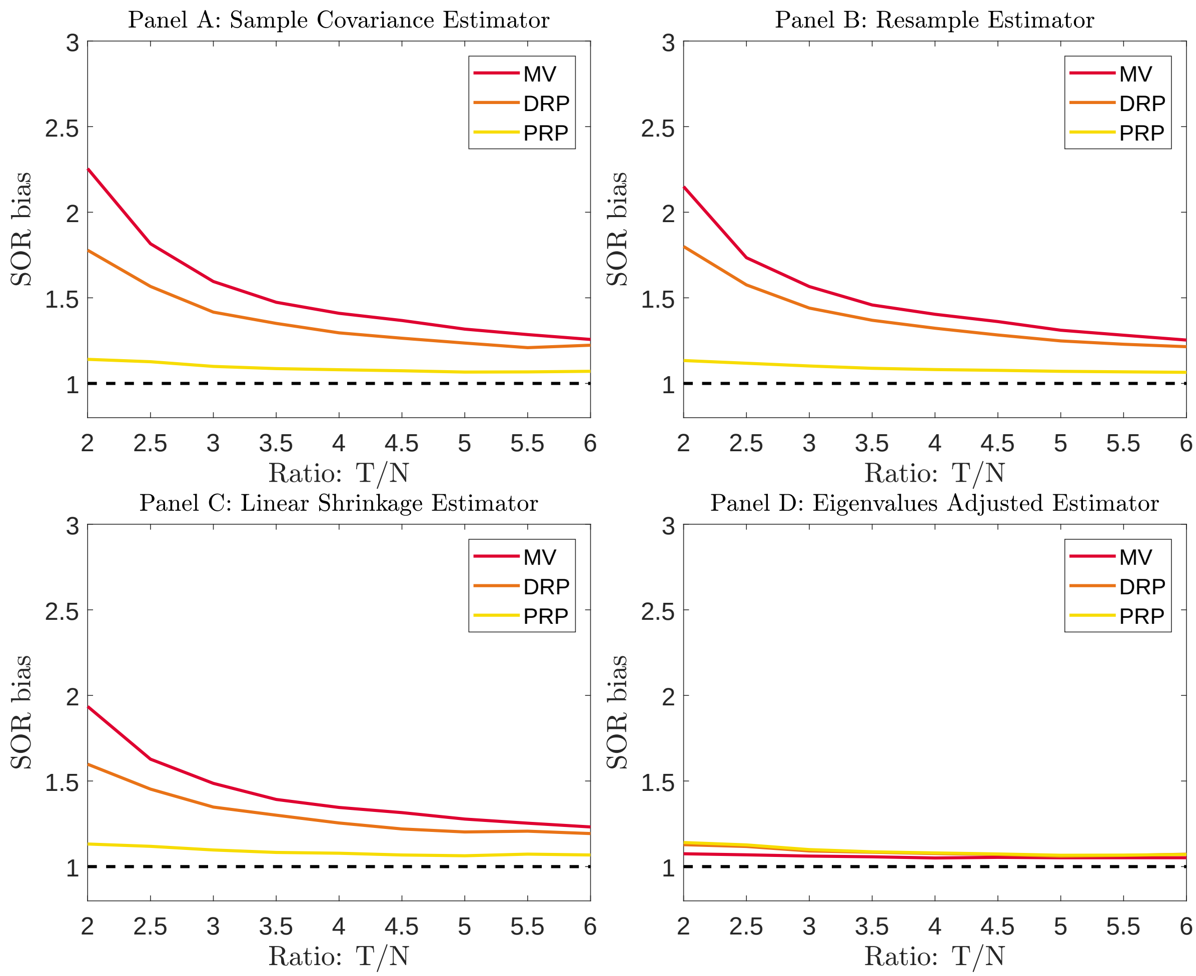


Figure 5. SOR Bias under Risk Mitigation Methodologies for Varying T/N Ratios
The figure shows the second-order risk bias (calculated via SOR bias statistic) for the MV portfolio, the DRP portfolio and the PRP portfolio under varying ratios of the number of daily returns (T) considered in-sample to the number of industry portfolios (N). The number of industry portfolios is kept constant at 10. Consequently, a ratio $T/N = 3$ corresponds to the case of 10 industry portfolios and 30 daily in-sample observations. Panel A displays the SOR bias of strategies constructed on the sample covariance matrix. Panel B, C, and D reports the SOR bias conditional on strategies being derived from the corresponding estimator of the covariance matrix: bootstrapping for panel B, linear shrinkage for panel C, and eigenvalue-adjustment for panel D. Results are derived with the asset universe described in Table 1 and results are generated using data ranging from April 2007 to March 2017.



The results displayed are consistent with the theory highlighted in Section 2. The MV exhibits the highest SOR bias, which increases with a decreasing number of observations. The DRP has a lower SOR bias than the MV. The PRP is unbiased, independently of the number of in-sample observations considered. For the MV and DRP portfolios, the figure also reports the SOR bias after application of the correction highlighted in equation (7) and equation (19), respectively. These appear close to 1, as expected. Panels A to D report the same analysis and vary in the choice of the estimate for the input covariance matrix to portfolio optimization. Panel A reports the results derived from the sample covariance matrix, Panel B repeats the same analysis averaged over multiple bootstrapped covariance matrices, Panel C shows the SOR bias derived using the linear shrinkage estimator, whereas in Panel D the sample covariance matrix used is modified by the eigenvalue adjustment. From this figure we observe how the effect of risk mitigation is homogeneous and proportional to the SOR bias over the T/N ratio.

Figure 4 is derived similarly to Figure 5 (looking at the same T/N ratios) but with a varying number of Industry Portfolios. CRSP provides alternative groupings of the same U.S. equities in Industry Portfolios at various granularities. Panels A to D in Figure 4 are derived based on CRSP groupings of 5, 10, 30, and 48 Industry Portfolios respectively. The number of Industry Portfolios considered seems to have a limited effect on the resulting SOR bias across strategies. Slightly higher figures are observed for lower numbers of assets and lower numbers of observations, most probably a result of noise rather than a deviation from the theory highlighted in Section 2.

4 Conclusion

In this paper we have provided theoretical and empirical evidence on the contribution of second-order risk to realized volatility for alternative risk parity strategies. In particular, we demonstrate that alternative risk parity strategies, such as diversified and principal risk parity, are significantly less sensitive to second-order risk than the classical minimum variance portfolio. In this regard, an adequate allocation of the risk budget along uncorrelated risk sources mitigates potential SOR biases, e.g., by allocating away from lower eigenvalue portfolios, or by relying more on the correlation structure than on the estimates of the eigenvalues in portfolio construction. Taking this insight to an extreme, we show how the principal risk parity strategy which attaches equal weights to uncorrelated risk sources exhibits low to no SOR bias. Additionally, we provide empirical evidence for the eigenvalue adjustment being the most effective in correcting for the SOR bias.

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5 Appendix

For the expression

$$\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1/2}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1/2}\middle|\hat{\mathbf{U}}\right],$$

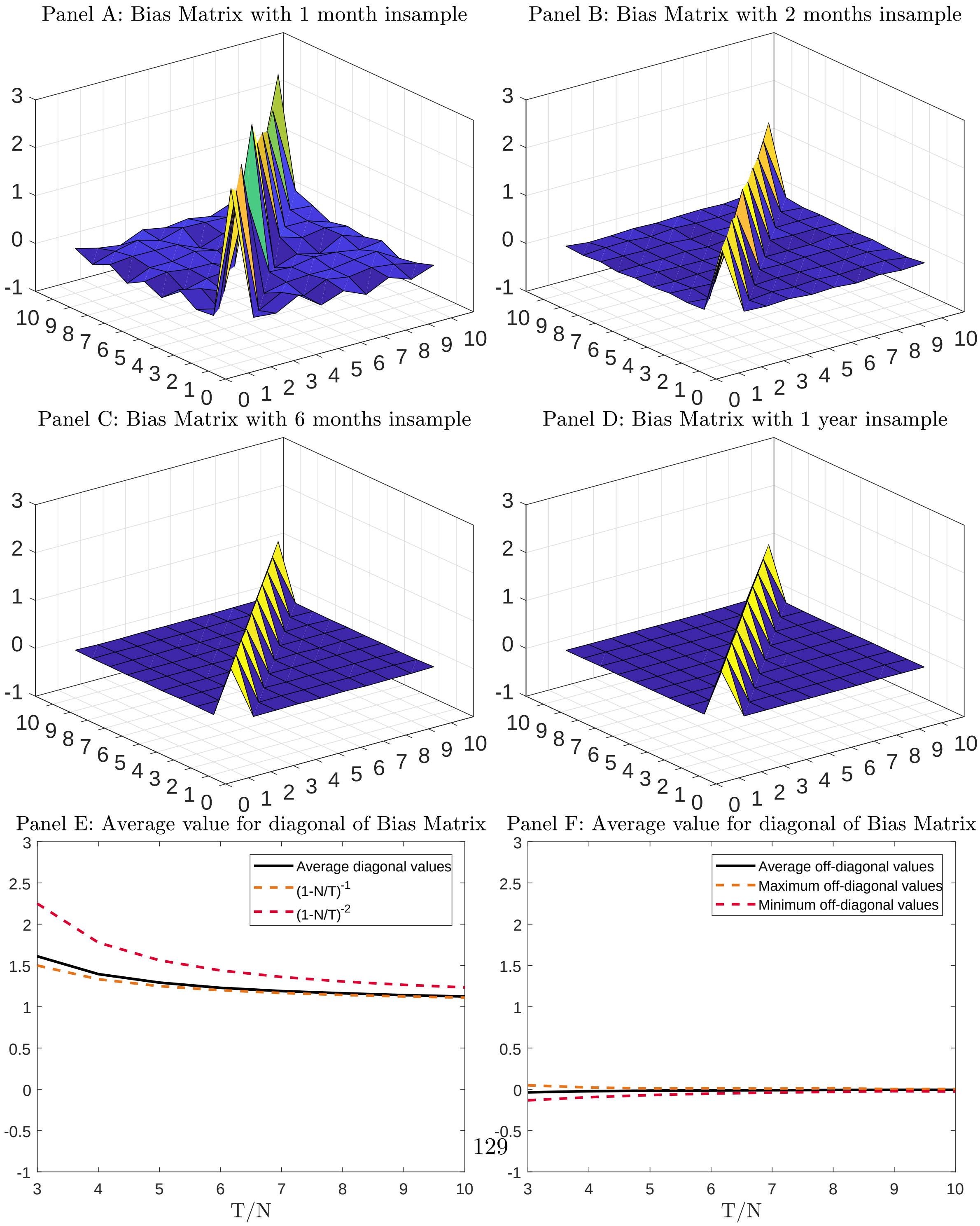
which we dub the "Bias Matrix," we cannot derive an analytical expression. Thus, we verify the validity of our approximation (C),

$$\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1/2}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1/2}\middle|\hat{\mathbf{U}}\right] \overset{(C)}{\simeq} \left(1-\frac{N}{T}\right)^{-1}\mathbb{E}\left[\mathbf{1}'\hat{\mathbf{U}}'\hat{\mathbf{U}}\mathbf{1}\right] = \left(1-\frac{N}{T}\right)^{-1}N, \tag{38}$$

via a Monte Carlo simulation. The derivation is done as follows. First a set of T asset returns for each of N assets is selected and the corresponding sample covariance matrix is calculated. This same matrix is assumed to be the true covariance matrix $\mathbf{\Omega}$. Second, this matrix is used to simulate a set of jointly normally distributed asset returns with mean 0 and covariance matrix $\mathbf{\Omega}$. For the simulated set of asset returns, the sample covariance $\hat{\mathbf{\Omega}}$, as well as the term $\hat{\mathbf{\Omega}}^{-1/2}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1/2}$ are calculated. This procedure is repeated 100,000 times, and the average of $\hat{\mathbf{\Omega}}^{-1/2}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1/2}$ gives an approximation for $\mathbb{E}\left[\hat{\mathbf{\Omega}}^{-1/2}\mathbf{\Omega}\hat{\mathbf{\Omega}}^{-1/2}\middle|\hat{\mathbf{U}}\right]$.

Figure 6. Monte Carlo Simulation: Estimation of DRP "Bias Matrix"

The figure shows the "Bias Matrix" of the Diversified Risk Parity (DRP) portfolio constructed via Monte Carlo simulation from the ten industry portfolios listed in Table 1. Panel A uses an in-sample time window of one month (corresponding to 21 daily returns). In Panels B, C, and D, the size of the in-sample time window increases to two, six, and twelve months, respectively. Panel E reports the average diagonal value of the Bias Matrix as a function of the ratio T/N. Panel F reports the average, maximum and minimum off-diagonal elements of the Bias Matrix as a function of the ratio T/N. Results are derived using data from April 2016 to March 2017.



Part V

Curriculum Vitae

Simone Bernardi

Curriculum Vitae

"Success is the ability to go from failure to failure without losing your enthusiasm." – Winston Churchill

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